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ADAPTIVE STOCHASTIC CONTROL FOR LINEAR SYSTEMS. PART II: ASYMPTOTIC PROPERTIES AND SIMULATION RESULTS

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PART II: ASYMPTOTIC PROPERTIES AND SIMULATION RESULTS*

by

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8. ASYMPTOTIC BEHAVIOR

In this section, we shall study the asymptotic behavior of the identification equations. The results will allow us to consider the problem of controlling the system $\mathcal{S}$ over an infinite time interval $(N \to \infty)$.

The main theoretical results will be stated; the proofs are given in Appendix C.

Definition 8.1: $\{(\underline{A}(k), \underline{C}(k))\}_{k=0}^{\infty}$ is said to be completely observable of index ν at k if the observation matrix

$$\underline{M}_{A,C}(k,\nu) = [\underline{C}'(k) \vdots \underline{\phi}_A'(k,k)\underline{C}'(k+1) \vdots \cdots \vdots \underline{\phi}_A'(k+\nu-2,k)\underline{C}'(k+\nu-1)] \tag{8.1}$$

is of full rank n. $\{(\underline{A}(k),\underline{C}(k))\}_{k=0}^{\infty}$ is said to be uniformly completely observable of index ν if the pair is completely observable of index ν for all $k = 0, 1, \ldots$.

Theorem 8.2: Let $\{(\underline{A}(k),\underline{C}(k))\}_{k=0}^{\infty}$ be uniformly completely observable of index ν, and suppose that $\underline{A}(k)$, $\underline{G}(k)$ are nonsingular, $k = 0, 1, \ldots$. If $u(k) \neq 0$, $k = 0, 1, \ldots$, then $\{(\underline{\tilde{A}}(k,u(k)),\underline{\tilde{C}}(k)\}_{k=0}^{\infty}$ is uniformly completely observable of index ν', $\nu' \leq 2\nu$.

Corollary 8.3: Let $\underline{A}(k)$, $\underline{G}(k)$ be bounded and nonsingular. If $\{(\underline{A}(k),\underline{C}(k))\}_{k=0}^{\infty}$ is uniformly completely observable of index ν, the error covariance matrix, $\underline{\Sigma}(k|k,U(0,k-1))$ which satisfies (4.21) to (4.23), will remain bounded for all $k = 0, 1, \ldots$ where $u(k)$ is any bounded but nonzero control for all $k = 0, 1, \ldots$.

Lemma 8.4: Suppose that $\underline{G}(k)$ satisfies

$$\underline{G}(k)\underline{B}\,\underline{G}'(k) \le \underline{B} \quad ; \quad \underline{B} \in M_{nn} \; , \; \underline{B} \ge \underline{0} \quad . \tag{8.2}$$

Let $\underline{\gamma}(k) \equiv \underline{0}$, i.e., there is no driving noise in the gain dynamics, then for any control sequence, we have

$$\underline{\Sigma}_b(k+1|k+1,U(0,k)) \le \underline{\Sigma}_b(k|k,U(0,k)) \quad . \tag{8.3}$$

We remark that Eq. (8.2) holds when $\underline{G}(k) = \underline{I}$ for all k, i.e., the unknown parameter vector $\underline{b}$ is constant.

An immediate consequence of lemma 8.4 is that if (8.2) is true and $\underline{\gamma}(k) \equiv \underline{0}$, then there exists $\underline{\Sigma}_b$ such that

$$\lim_{k\to\infty} \underline{\Sigma}_b(k|k,U(0,k-1)) = \underline{\Sigma}_b \tag{8.4}$$

Note that (8.4) is true independent of the observability of $\{(\underline{A}(k),\underline{C}(k))\}_{k=0}^{\infty}$. In the following theorem, we shall give sufficient conditions under which $\underline{\Sigma}_b \equiv \underline{0}$.

Theorem 8.5. (Main result): Let $\underline{\gamma}(k) = \underline{0}$, $\underline{A}(k)$, $\underline{G}(k)$ be bounded and nonsingular and $\underline{G}(k)$ satisfies (8.2)), k = 0, 1, ... If $\{(\underline{A}(k),\underline{C}(k))\}_{k=0}^{\infty}$ is uniformly completely observable of index ν and u(k) is any bounded but nonzero control for k = 0, 1, ..., then

$$\lim_{k\to\infty} \underline{\Sigma}_b(k|k,U(0,k-1)) = \underline{0} \quad . \tag{8.5}$$

Theorem 8.5 can be extended to the case where u(k) is bounded but nonzero control for all but a finite number of k's. Since $\underline{\Sigma}(k|k,U(0,k-1)) \ge \underline{0}$,

(8.5) also implies

$$\lim_{k\to\infty} \underline{\Sigma}_{xb}(k|k,U(0,k-1)) \to \underline{0} \tag{8.6}$$

if the conditions for theorem 8.5 hold.

Let us consider an observable system S, (2.1), the gain parameters are assumed to be unknown and satisfy

$$\underline{b}(k+1) = \underline{G}(k)\underline{b}(k) \tag{8.7}$$

with $\underline{G}(k)$ satisfying (8.2). Assume that we want to control the system S over an interval $N < \infty$. In the beginning, the modified weighting on the control is high, and thus in general, the control magnitude will be low at the beginning. Thus, the trajectory of the overall control system would be pretty much the same as the input-free trajectory of the system S. If the matrix A(k) is exponentially stable, the true state of the system will evolve toward zero by using negligibly small control magnitudes (even zero). The result is that little effort of the input, $\{u(k)\}_{k=0}^{N-1}$ is spent for control and identification purposes. We would expect that the estimated parameters will hardly converge to the true parameters, $\underline{b}(k)$. On the other hand if A(k) is not exponentially stable, then the true state of the overall system will diverge. This diverging phenomenon will be noticed by the identifier, thus resulting in a high control magnitude because of (5.1). Since little is initially known about the gain parameters, the high magnitude control will be utilized mainly for identification purposes. Therefore the control will be ~~kept~~ bounded away from zero as long as exact identification of $\underline{b}(k)$ has ~~not been~~ obtained. Using theorem 8.5, we predict that for unstable systems the estimated parameters of $\underline{b}(k)$ will converge to the true gain parameters before the control magnitude goes to zero. This is also borne out by the simulation results.

Analytical studies of the convergence rate of the O.L.F.O. system are not yet ~~available~~. From the above discussion, we may predict roughly that the ~~convergence-rate~~ for unstable system will be relatively fast and the convergence-rate for stable system will be very slow.

Finally, we shall discuss some interesting implications of theorem 8.5. . Consider an observable system S, (2.1), , with unknown gain parameters satisfying (8.7)) and with $\underline{G}(k)$ satisfies (8.6).). Let $\phi_k(\hat{\underline{x}}(k|k,\hat{\underline{b}}(k|k),\underline{\Sigma}_b(k|k,U(0,k-1))$ be any ad-hoc control law which is "placed" after the identifier and with the following properties $(k \geq 0)$;

1) $\phi_k(\cdot,\cdot,\cdot)$: $R^n \times R^n \times M_{nn} \to R$
2) $\phi_k(\underline{x},\underline{b},\underline{\Sigma}) \neq \underline{0}$, $\underline{x} \in R^n$, $\underline{b} \in R^n$, $\underline{\Sigma} \in M_{nn}$, $\underline{x} \neq \underline{0}$; $\underline{\Sigma} \neq \underline{0}$
3) $\phi_k(\underline{x},\underline{b},\underline{0}) = -(h(k)+\underline{b}'(k)\underline{K}(k+1)\underline{b}(k))^{-1}\underline{b}'(k)\underline{K}(k+1)\underline{A}(k)\underline{x}$;
$\underline{x} \in R^n$, $\underline{b} \in R^n$

From condition 2, we see that $\underline{\Sigma}_b(k|k,U(0,k-1)) \to \underline{0}$ as $k \to \infty$ and so from condition 3, the ad-hoc control scheme will converge to the optimal control strategy when the full dynamics become known. This indicates that the ad-hoc scheme $\phi_k(\hat{\underline{x}}(k|k),\hat{\underline{b}}(k|k),\underline{\Sigma}_b(k|k,U(0,k-1))$ can provide reasonable results.

9. REMARKS

Vector Control

In our investigation, we assumed that the control is scalar. However, the approach can be extended in a straightforward conceptual manner to the vector control case. First, a set of identification equations is derived which will generate the estimate of the current state, the current estimate of the unknown gain matrix and the different cross-error-covariance matrices. An open-loop control problem is formulated as in Section 4, equations (4.20) to (4.31) and discrete matrix minimum principle is used to obtain the extremal solution. The results will be similar to those of scalar control case. However, the equations in the vector control case will look and be more complicated.

Control Over Infinite Interval

Let us consider the problem of controlling the system S, which is time invariant and with an unknown constant gain vector $\underline{b}$, over an infinite interval, i.e., $N \to \infty$. To obtain a feasible solution we suggest the window-shifting approach. Assume that at all times, we have N more steps to control, thus at all times we solve an open-loop control problem over an interval of N steps. This approach is motivated by computational considerations and the theoretical results derived in Section 8.

We note that in the O.L.F.O. approach, we have to re-solve the open-loop control problem at every time k so as to adjust the control scheme accordingly In our case, we have to compute $\tilde{\underline{K}}(k|k)$ in a backward direction starting from the terminal time N to k for each k. If N is very large, this requires a large computation time. From a computational standpoint, we would like to "cut back" the terminal time. Conceptually, in trying to control over an infinite time period, the controller looks into all future effects caused by present action, and decides on the optimum move for the next step. The window-shifting approach suggests that instead of looking at all future effects, the controller looks at only near future effects caused by present actions and decides on suboptimal moves. One may view such an approach as a "~~short~~ term adaptive scheme." Note also that we can adjust the "window width" ~~according~~ to our computational capability. At all times, we need only to solve for $\underline{\tilde{K}}(k|k)$ in a backward direction starting from N + k to k. Thus, from a conceptual and a computational point of view, such an approach may be desirable.

Assume that the time invariant system $\mathcal{S}$ being controlled is observable and controllable. If $\underline{b}$ is known exactly, then if we consider control over an infinite time period, the optimal feedback gain is constant and is given by

$$\underline{\phi} = -(h + \underline{b}'\underline{K}\,\underline{b})^{-1}\underline{b}'\underline{K}\,\underline{A} \tag{9.1}$$

where $\underline{K}$ is given by the steady state solution of

$$\underline{K}_{i+1} = \underline{A}'(\underline{K}_i - \underline{K}_i\underline{b}(h + \underline{b}'\underline{K}_i\underline{b})^{-1}\underline{b}'\underline{K}_i)\underline{A} + \underline{W} \quad ; \quad \underline{K}_o = \underline{F} \tag{9.2}$$

Let N be the integer such that for n ≥ N,

$$||\underline{K}_n - \underline{K}_{n-1}|| \le \varepsilon \quad ; \quad \varepsilon > 0 \quad . \tag{9.3}$$

Such an integer N can be found experimentally off-line. Adjust the window width equal to N, and apply the window-shifting approach.

By theorem 8.5, the estimate in $\underline{b}$ will converge asymptotically, and so when $\hat{\underline{b}}(k|k,U^*(0,k-1) \to \underline{b}$, we have

$$\underline{\tilde{K}}(k|k) \to \begin{bmatrix} \underline{K}(k,N+k;\underline{F}) & \vdots & \underline{0} \\ \cdots & & \cdots \\ \underline{0} & \vdots & \underline{0} \end{bmatrix}$$

where $\underline{K}(k,N+k;\underline{F})$ satisfies

$$\underline{K}(k,N+k;\underline{F}) = \underline{A}'(\underline{K}(k+1,N+k;\underline{F}) - \underline{K}(k+1,N+k;\underline{F})\underline{b}(h + \underline{b}'\underline{K}(k+1,N+k;\underline{F})\underline{b})^{-1}\cdot$$

$$\underline{b}'\underline{K}(k+1,N+k;\underline{F}))\underline{A} + \underline{W} \quad ; \quad \underline{K}(N+k,N+k;\underline{F}) = \underline{F} \tag{9.4}$$

and

$$u^*(k|k) \to \underline{\phi}(k)\underline{\hat{x}}^{\sigma}(k\ k) = -(h + \underline{b}'\underline{K}(k,N+k;\underline{F})\underline{b})^{-1}\underline{b}'\underline{K}(k,N+k;\underline{F})\underline{A}\,\underline{\hat{x}}^{o}(k|k) \tag{9.5}$$

(See discussion in Section 6.) Comparing (9.2) and (9.4), we note that

$$\underline{K}(k,N+k;\underline{F}) = \underline{K}_N \simeq \underline{K} \quad . \tag{9.6}$$

Thus asymptotically, the time varying adaptive system tends to be a time invariant control system.

10. NUMERICAL EXAMPLES

In the previous sections, we have studied theoretically the adaptive control of a discrete time linear system with an unknown gain vector. An adaptive system was derived using the O.L.F.O. approach, and the asymptotic behavior of the control system was discussed. There are still some important questions which have not been treated theoretically. For example, rates of convergence are, in general, of great interest, but this topic was not treated in detail. In this section we present simulation studies carried out for some specific third order systems. The main purpose for these studies is to provide quantitative results about rates of convergence and to test the validity of the qualitative conclusions of Section 6.

To enhance physical intuition, the discrete time systems were obtained by sampling a continuous-time system. In this case, the uncertainty of the $\underline{b}(k)$ vector is equivalent to uncertainty as

(a) To the number of zeros,

(b) The location of the zeroes in the S-plane, and

(c) The plant DC gain.

It is assumed that the pole locations are known.

Let us consider a stochastic continuous time-invariant linear system described by:

$$\dot{\underline{x}}_f(t) = \underline{A}_f\underline{x}_f(t) + \underline{b}_f u_f(t) + \underline{d}_f \xi_f(t) \quad ; \quad \underline{x}(0) \sim \mathcal{G}(\underline{0}, \underline{\Sigma}_{xo})$$
$$\underline{y}_f(t) = \underline{c}'\underline{x}_f(t) + \eta_f(t) \qquad\qquad \underline{b}_f \sim \mathcal{G}(\underline{0}, \underline{\Sigma}_{bo}) \tag{10.1}$$

where $\xi_f(t)$ is a scalar driving white Gaussian noise, $\eta_f(t)$ is the

scalar observation white Gaussian noise. The statistical laws of $\xi_f(t)$ and $\eta_f(t)$ are assumed to be known:

$$\int_{t_1}^{t_2} \xi_f(t)dt \sim \mathcal{G}\left(0, \int_{t_1}^{t_2} r\ dt\right) \tag{10.2}$$

$$\int_{t_1}^{t_2} \eta_f(t)dt \sim \mathcal{G}\left(0, \int_{t_1}^{t_2} q\ dt\right) \tag{10.3}$$

From (10.1), we have

$$x_f(t) = e^{\underline{A}_f t}\underline{x}_f(0) + \int_0^t e^{\underline{A}_f(t-\tau)}\underline{b}_f u_f(\tau)\,d\tau + \int_0^t r^{\underline{A}_f(t-\tau)}\underline{d}_f\xi_f(\tau)d\tau \tag{10.4}$$

Assume that we take observations only at discrete instants of time $t = \Delta$, 2Δ, 3Δ ...; Δ is assumed to be small such that $u(t) = u(k\Delta)$, $\xi(t) = \xi(k\Delta)$, $t \in [k\Delta, (k+1)\Delta]$:

$$\begin{aligned}\underline{x}_f\ (k+1)\Delta = e^{\underline{A}_f\Delta}\Big[&e^{\underline{A}_f(k\Delta)}\underline{x}_f(0) + \int_0^{k\Delta} e^{\underline{A}_f(k\Delta-\tau)}\underline{b}_f\,u_f(\tau)d\tau \\ &+ \int_0^{k\Delta} e^{\underline{A}_f(k\Delta-\tau)}\underline{d}_f\,\xi_f(\tau)d\tau\Big] \\ &+ \int_0^{\Delta} e^{\underline{A}_f\sigma}d\sigma \cdot \underline{b}_f\,u_f(k\Delta) + \int_0^{\Delta} e^{\underline{A}_f\sigma}d\sigma \cdot \underline{d}_f\,\xi_f(k\Delta)\end{aligned} \tag{10.5}$$

Defining

$$\begin{aligned}&\underline{x}(k) \triangleq \underline{x}_f(k\Delta) \ ; \ \underline{A} \triangleq e^{\underline{A}_f\Delta} \ ; \ \underline{b} \triangleq \int_0^{\Delta} e^{\underline{A}_f\sigma}d\sigma \cdot \underline{b}_f \\ &\underline{d} \triangleq \int_0^{\Delta} e^{\underline{A}_f\sigma}d\sigma \cdot \underline{d}_f \ ; \ \xi(k) \triangleq \xi_f(k\Delta) \ ; \ u(k) \triangleq u_f(k\Delta),\end{aligned} \tag{10.6}$$

(10.5) becomes

$$\underline{x}(k+1) = \underline{A}\,\underline{x}(k) + \underline{b}\,u(k) + \underline{d}\xi\,(k) \quad ; \quad \underline{x}(0) \sim \mathcal{G}(\underline{0}, \underline{\Sigma}_{xo}) \tag{10.7}$$

Defining

$$y(k) \overset{\Delta}{=} y_f(k\Delta) \quad ; \quad \eta(k) \overset{\Delta}{=} \eta_f(k\Delta) \tag{10.8}$$

the observation sequence is

$$y(k) = \underline{c}'\underline{x}(k) + \eta(k) \tag{10.9}$$

The statistical laws of $\xi(k)$, $\eta(k)$ are

$$\xi(k) \sim G(0, r\Delta) \tag{10.10}$$

$$\eta(k) \sim G(0, q\Delta) \tag{10.11}$$

The gain vector is assumed to be unknown but constant, therefore the equation for the unknown gain is

$$\underline{b}(k+1) = \underline{b}(k) \quad ; \quad \underline{b}(0) \sim G(\underline{b}_o, \underline{\Sigma}_{bo}) \tag{10.12}$$

We can now apply the results of previous sections to equations (10.6), (10.9), (10.11), and (10.12).

A computer program was designed which operates as follows:

(1) Read in $\underline{A}_f$, $\underline{b}_f$, $\underline{c}$, $\underline{d}_f$, r, q, , $\underline{x}_o$, $\underline{b}_o$, the sampling interval Δ, the final time N and the different weightings $\underline{W}$, h, $\underline{F}$, and covariances $\underline{\Sigma}_{xo}$, $\underline{\Sigma}_{bo}$.

(2) A subroutine, which was developed by Levis[11], was used to convert the continuous version, (10.1), to the discrete time sample data version (10.6). The covariances of $\xi(k)$, $\eta(k)$ are computed using (10.11), (10.12).

(3) The true value of $\underline{x}(k)$ was recorded. Using a noise generating subroutine, a sample value of $y(k)$ was obtained. Assume that $\hat{\underline{x}}(k-1/k-1)$, $\hat{\underline{b}}(k-1/k-1)$ are recorded. A subroutine for the identification equations (4.19)-(4.23) was used to obtain the current estimates $\hat{\underline{x}}(k/k)$, $\hat{\underline{b}}(k/k)$, and the error covariance matrix $\underline{\Sigma}(k/k)$ recursively. These values were also recorded.

(4) A subroutine based on (5.1) - (5.10) was used to obtain the adaptive control $u^*(k)$.

(5) The control $u^*(k)$ was applied to the system (10.6), using a noise generating device to obtain a sample value of $\xi(k)$; then by (10.6), we obtained the value $\underline{x}(k+1)$.

(6) We advance $k \to k+1$ and repeat (3) through (5) until we get to the final time $k = N-1$.

The program was written in such a way that if we set $\hat{\underline{b}}(k/k) = \underline{b}$, and $\underline{\Sigma}_{bo} = \underline{0}$, then the procedures (3) through (6) will give us the truly optimal stochastic control when $\underline{b}$ is known. Using a plotting subroutine we can plot out the truly optimal trajectories vs. the O.L.F.O. trajectories; the true $\underline{b}$ vs. the estimated $\hat{\underline{b}}$, and optimal feedback gain vs. adaptive gain (it was noted that the adaptive control correction term will converge to zero quite fast), under the requirement that the same noise samples ($\xi(k)$, $\eta(k)$) were used for both the "known $\underline{b}$" and "unknown $\underline{b}$" cases. These plots provide us with qualitative understanding on the rate of convergence of the overall suboptimal O.L.F.O. control system, and the effects of unknown gains.

In all the computer simulations, unless otherwise mentioned, we set the values:

$$\Delta = 0.2 \text{ sec.}, \quad r = 0.05, \quad q = 0.45, \quad \underline{d}_f = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \underline{x}_o = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$
$$\underline{F} = \underline{I}_3, \quad \underline{W} = \underline{I}_3, \quad \underline{\Sigma}_t = \underline{\Sigma}_{xo} = 4\,\underline{I}_3, \quad \underline{c}' = [1\ 0\ 0] \qquad (10.13)$$

It is important to realize then that we deal with a third order system. The only measurement is that of the output, every 0.2 seconds. This sampled-data measurement is corrupted by white noise whose variance is $\rho = (0.45)(0.2) = 0.09$ (or R.M.S. value 0.3). The plant may have none, one, or two zeroes. We do not know how many there are or where. Hence, even though the poles are assumed known, the measurements are extremely meager since from the noisy measurement of one variable, one must estimate six (three state variables and three parameters that define the number and location of zeros). Furthermore, the open loop plant may be unstable.

Example 1: Unstable System

It is assumed that the continuous time system is described by

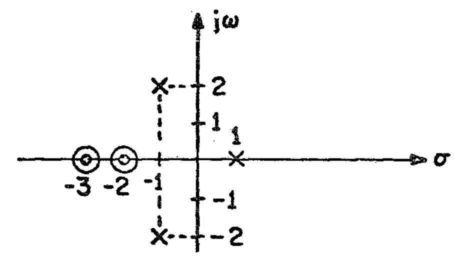
$$\underline{A}_f = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -3 & -1 \end{bmatrix}; \quad \underline{b}_f = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}; \quad h = 1; \quad \underline{x}_f(0) = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \qquad (10.14)$$

such a system has a true transfer function (see Fig. 2)

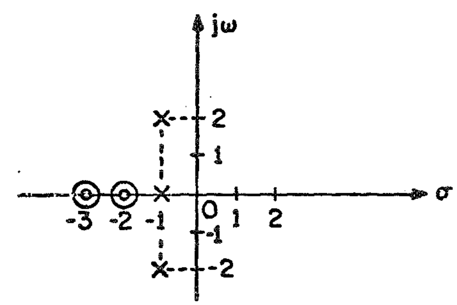
$$H_1(s) = \frac{(s+3)(s+2)}{(s-1)(s^2+2s+s)} \qquad (10.15)$$

Note that it has an unstable pole at $s = 1$. Initially, we set

$$\hat{\underline{b}}_f(0/0) = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \qquad (10.16)$$



POLE ZERO PATTERN FOR EXAMPLE 1: UNSTABLE SYSTEMS



POLE ZERO PATTERN FOR EXAMPLE 2: STABLE SYSTEM

FIGURE 2

i.e. we start with an initial guess that the system hasno zeroes. The final time is N = 40 (or 8 seconds).

Many computer runs have been made on the same system with different noise samples. The plots for one particular sample experiment, which represents a fairly average behavior, are shown in Figs. 3, 4, and 5. From the simulation data (which is not shown completely), we can obtain a rough idea about the behavior of the suboptimal O.L.F.O. control system.

From the simulations, it was found that in the beginning, the O.L.F.O. adaptive gain is approximately zero (Fig. 5) and the O.L.F.O. trajectory follows closely to the input-free trajectory (Fig. 3).

This agrees with the discussion in Section 6 regarding the effect of having an unstable system during the initial measurements ($\tilde{h}$ very large) in which little control is applied. The diverging phenomenon, due to the plant instability, is detected by the identifier; controls of considerably high magnitude are then applied for a few steps. This is indicated by the fact that there are sharp jumps in the state trajectories. The simulation showed that these jumps are not caused by bad noise samples, because the same phenomenon appears in different sample runs at approximately the same time interval. The high magnitude control serves mainly for identification purposes; this is revealed by the fact that at the next time unit, the estimate of b closely agrees with the true b (Fig. 4). As was predicted in Section 7, the O.L.F.O. adaptive gains do converge to the truly optimum gains (Fig. 4). The correction term vs. time is not shown in the figure, but simulation results indicate that the correction term goes to zero very rapidly after the identification of b is essentially complete.

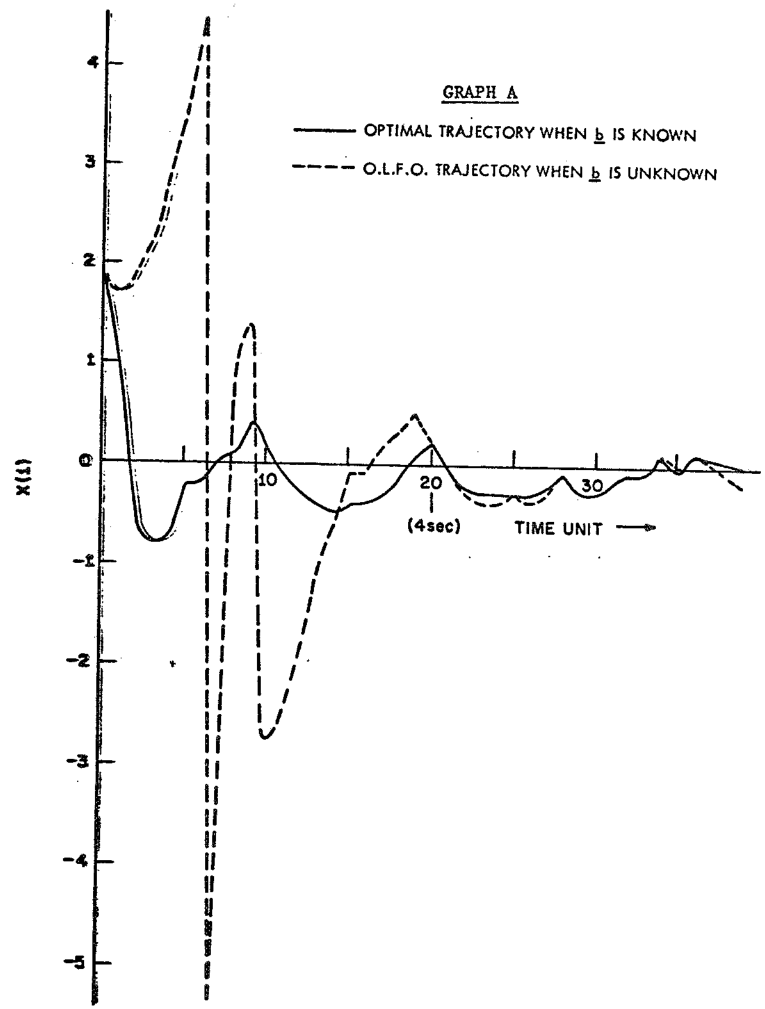


Fig. 3 COMPARISON BETWEEN THE OPTIMAL TRAJECTORY WHEN THE GAIN IS KNOWN AND THE O.L.F.O. TRAJECTORY ASSUMING THE GAIN IS UNKNOWN. THE SYSTEM BEING CONTROLLED IS UNSTABLE WITH SYSTEM FUNCTION $\frac{(S+3)\ (S+2)}{(S-1)\ (S^2+2S+5)}$.

THE SAMPLE NOISE IS THE SAME FOR BOTH CASES.

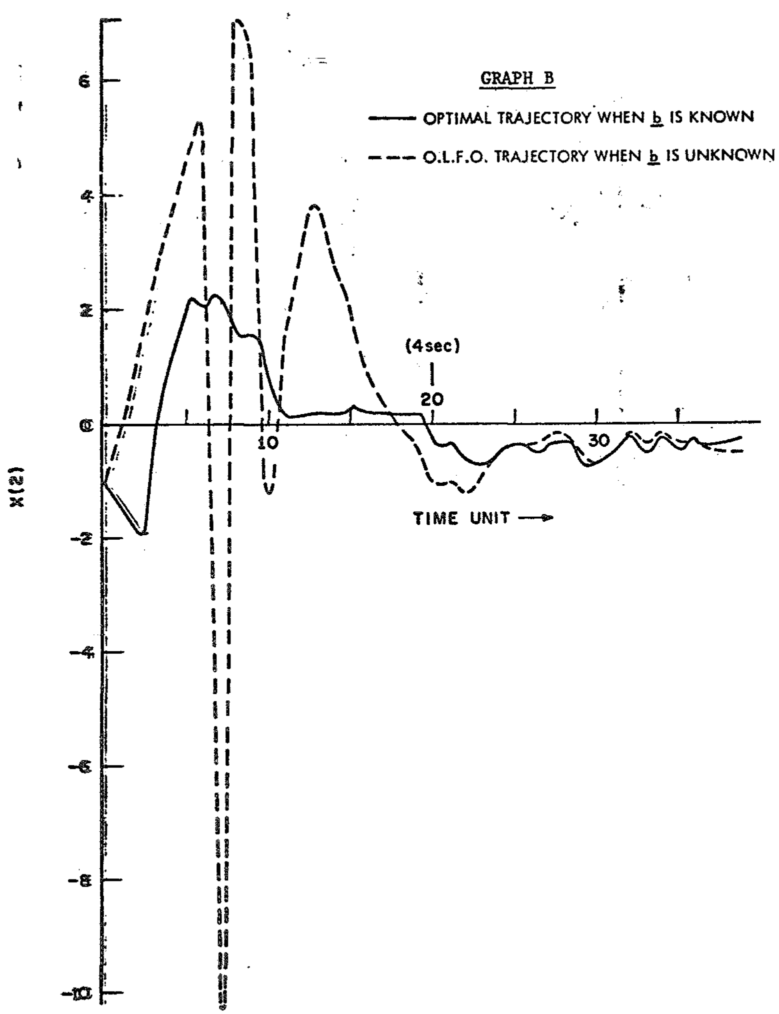


Fig. 3 (Continued)

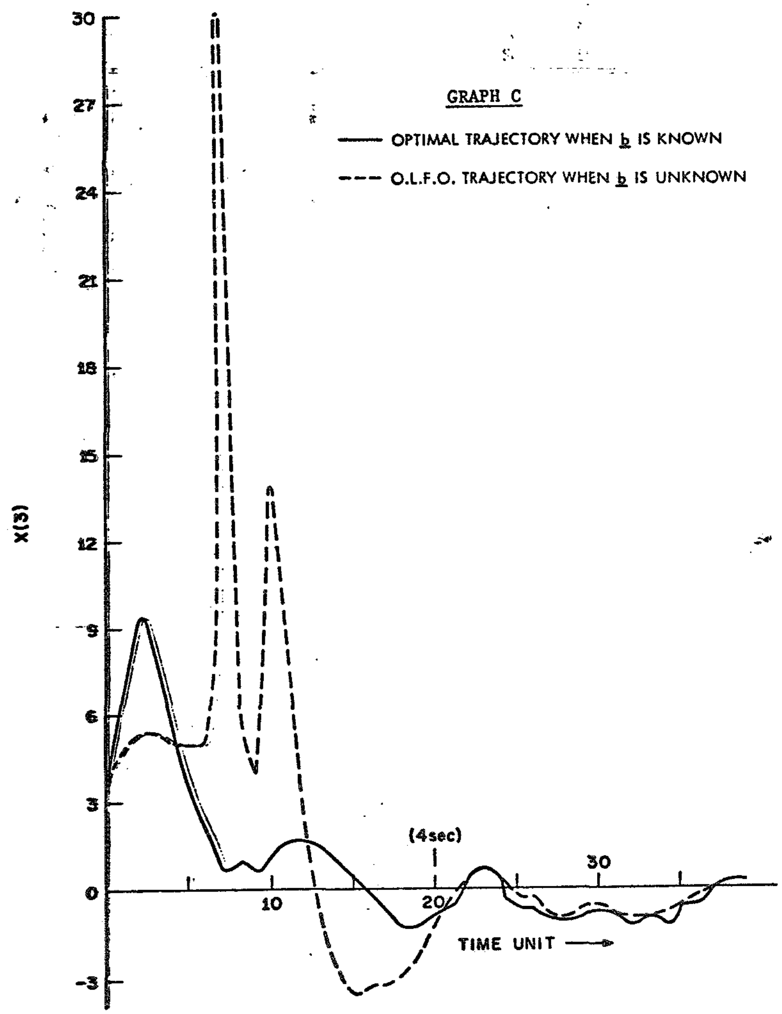


Fig. 3 (Continued)

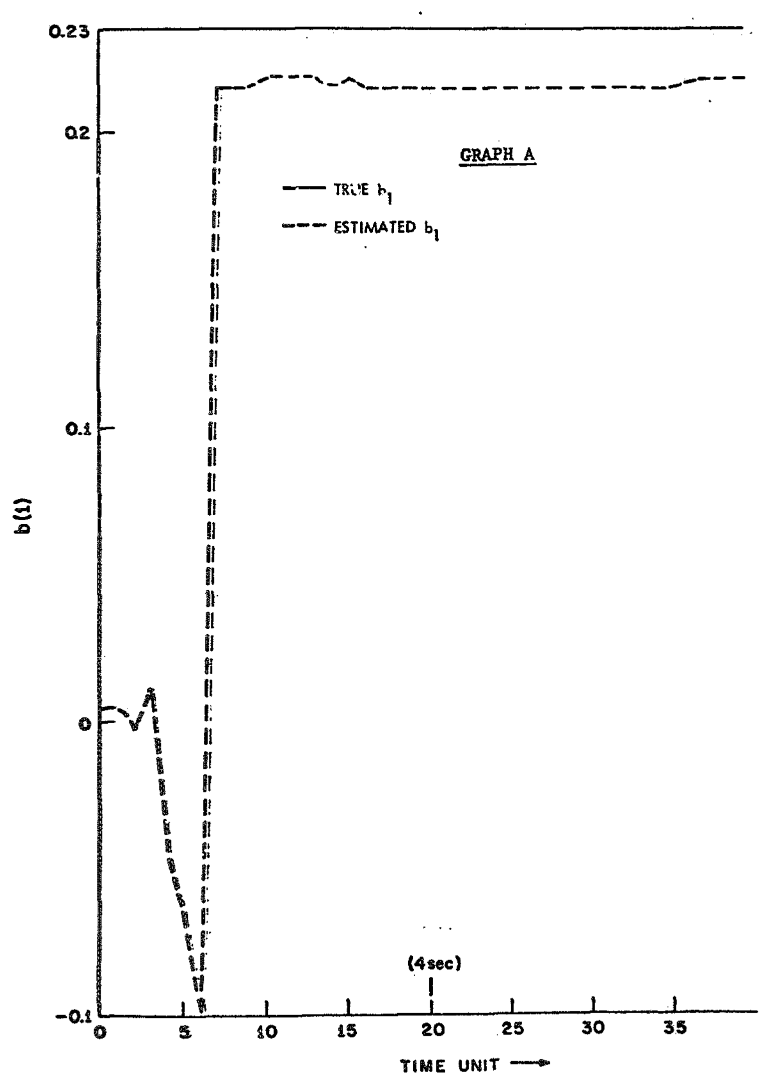


Fig. 4 ESTIMATE OF THE UNKNOWN GAIN VECTOR. THE SYSTEM BEING CONSIDERED HAS SYSTEM FUNCTION $\frac{(S+3)\ (S+2)}{(S-1)\ (S^2+2S+5)}$

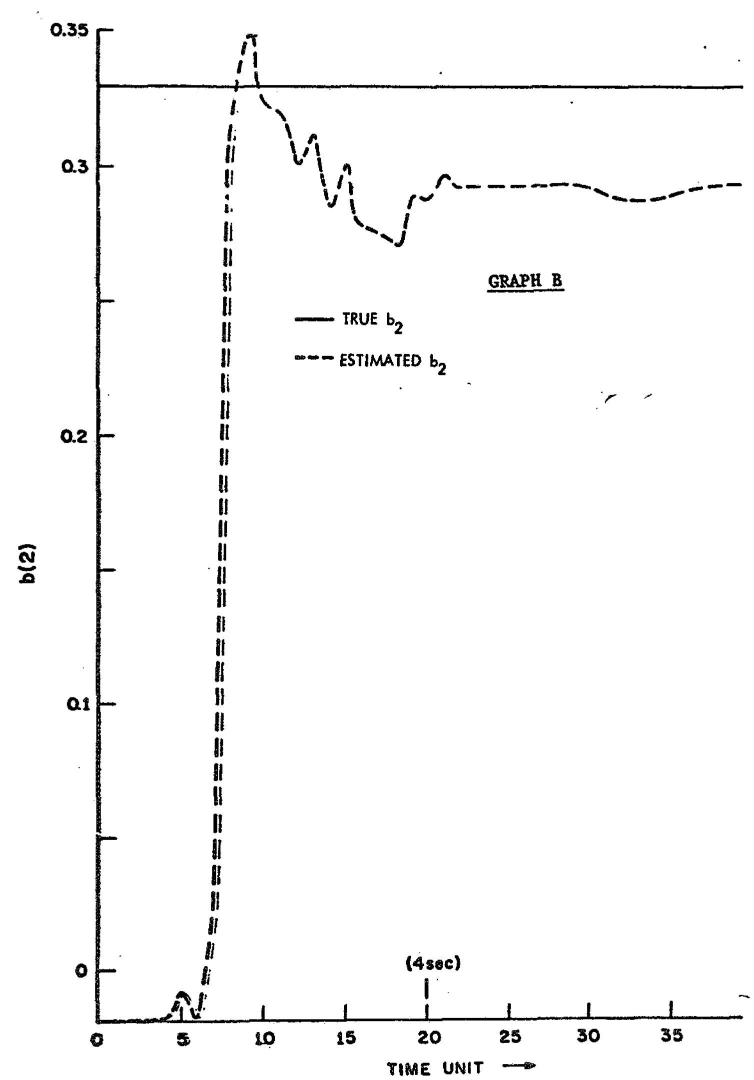


Fig. 4 (Continued)

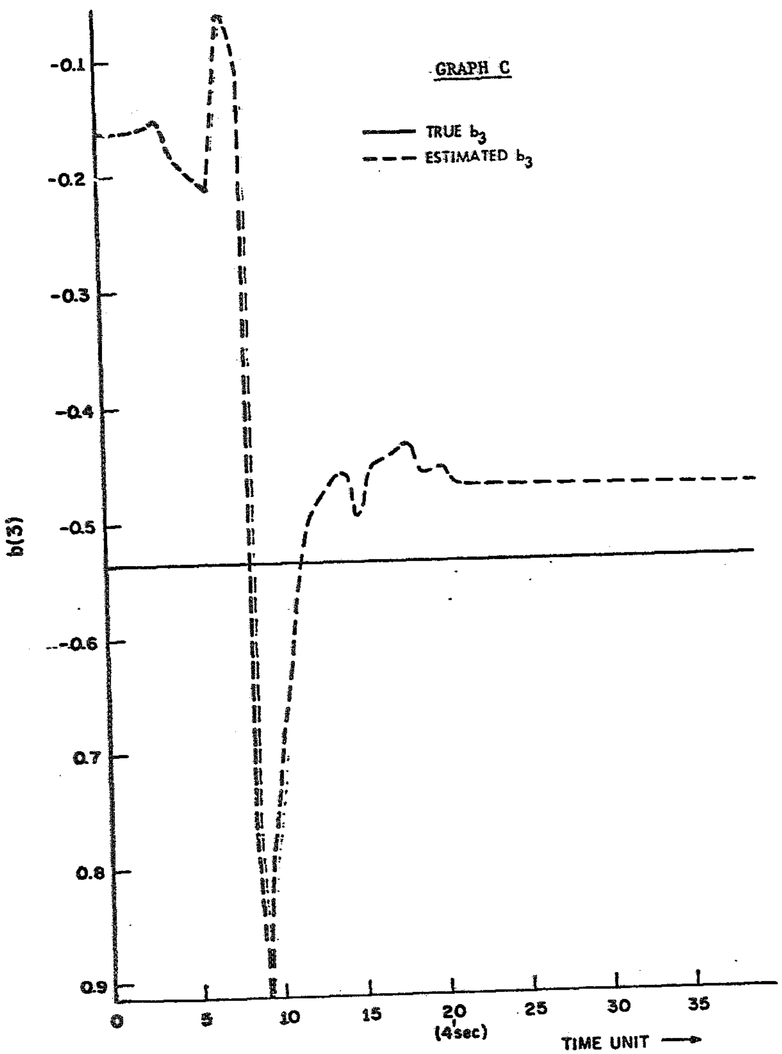


Fig. 4 (Continued)

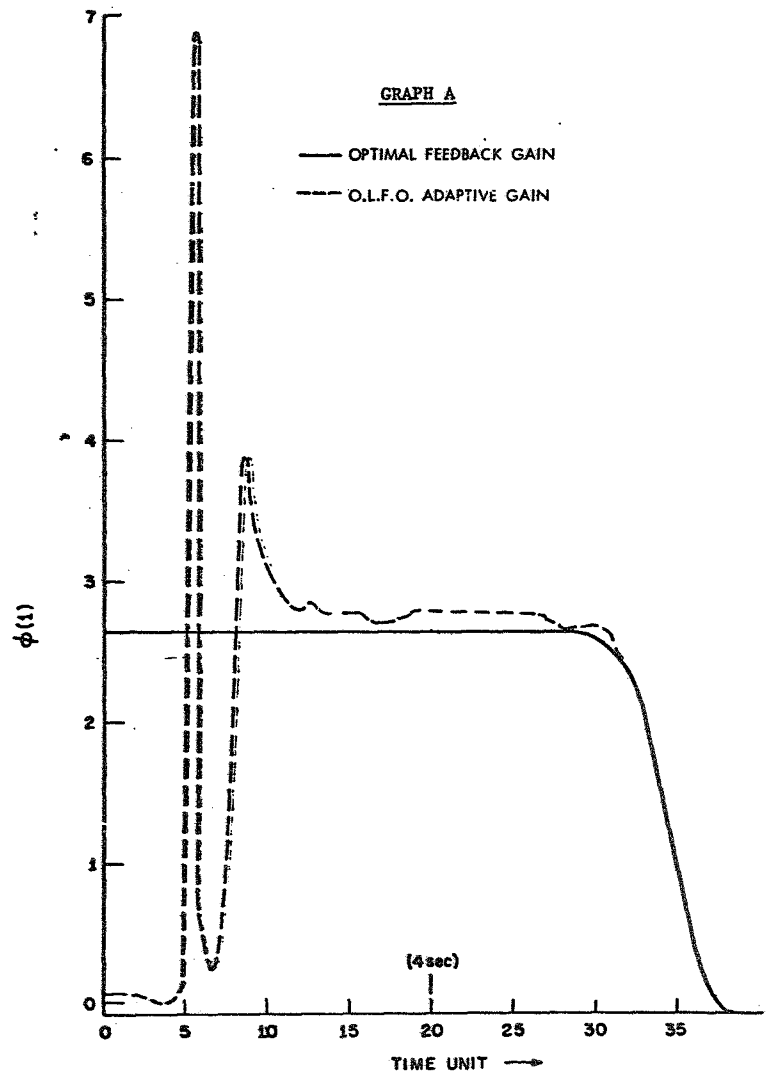


Fig. 5 COMPARISON BETWEEN THE OPTIMAL FEEDBACK GAINS AND THE ADAPTIVE O.L.F.O. GAINS. THE SYSTEM BEING CONSIDERED HAS SYSTEM FUNCTION $\frac{(S+3)\ (S+2)}{(S-1)\ (S^2+2S+5)}$

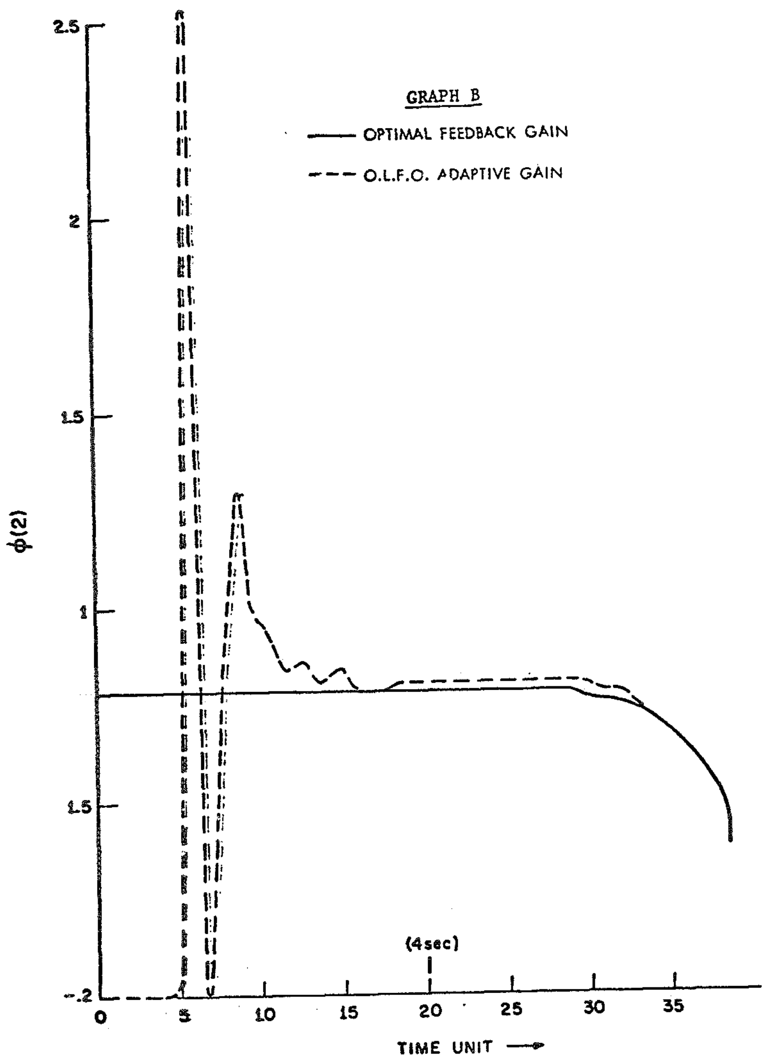


Fig. 5 (Continued)

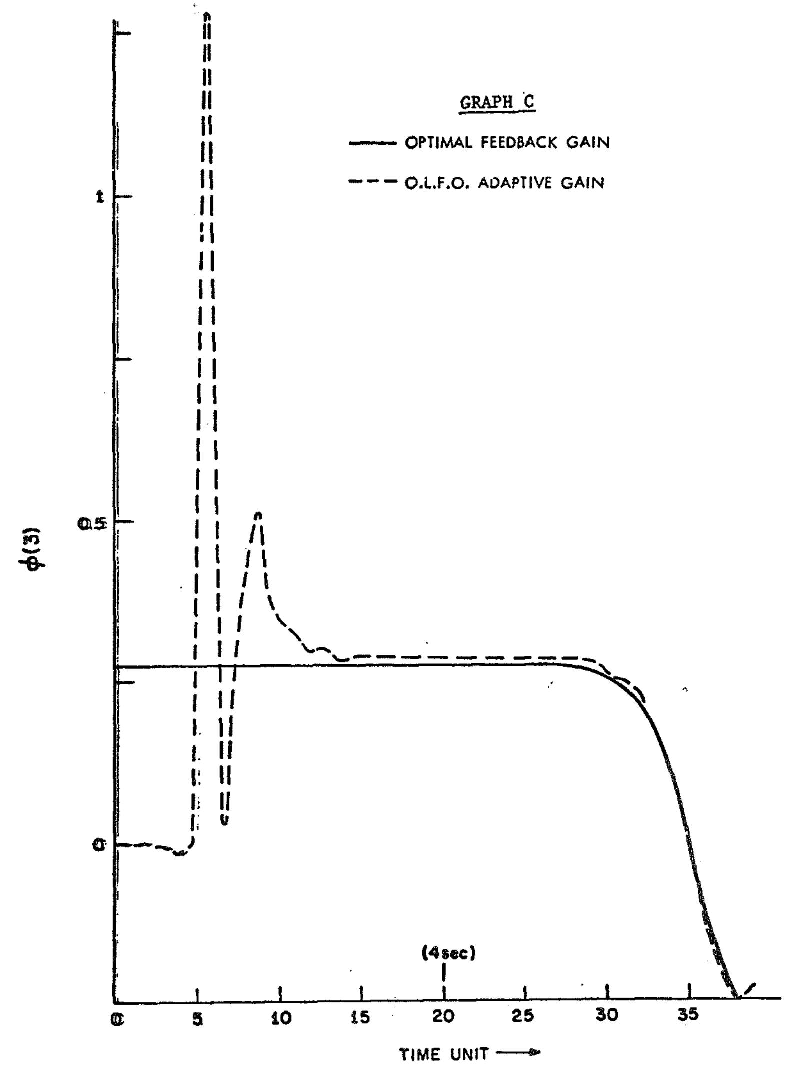


Fig. 5 (Continued)

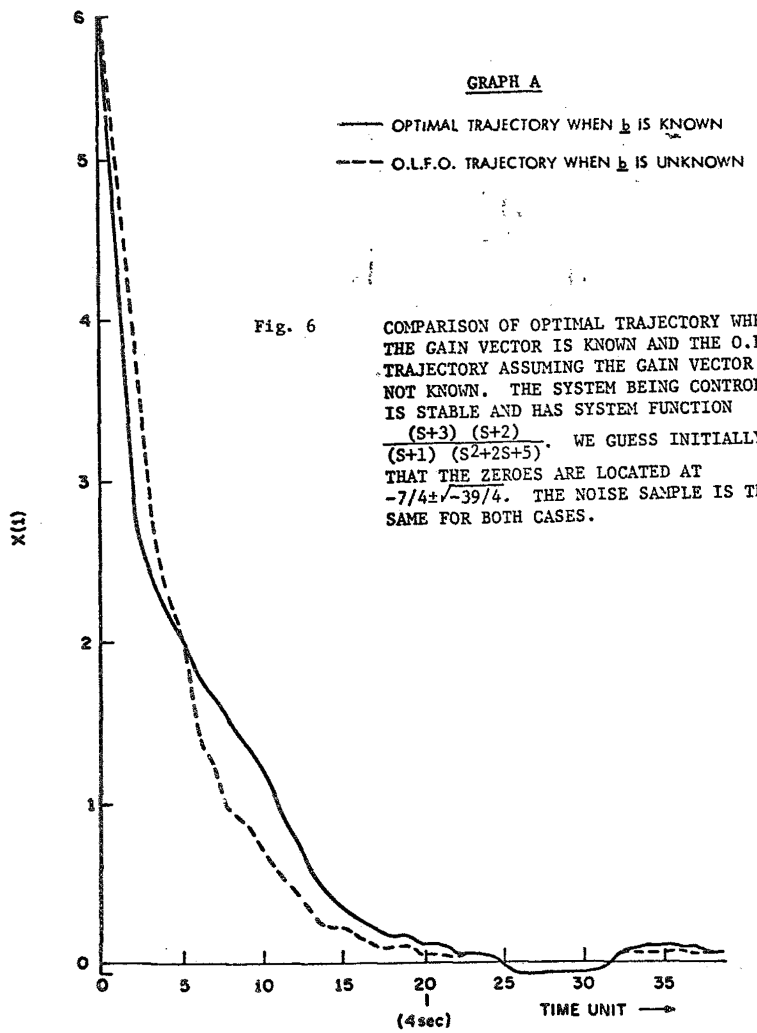


Fig. 6 COMPARISON OF OPTIMAL TRAJECTORY WHEN THE GAIN VECTOR IS KNOWN AND THE O.L.F.O. TRAJECTORY ASSUMING THE GAIN VECTOR IS NOT KNOWN. THE SYSTEM BEING CONTROLLED IS STABLE AND HAS SYSTEM FUNCTION $\frac{(S+3)\ (S+2)}{(S+1)\ (S^2+2S+5)}$. WE GUESS INITIALLY THAT THE ZEROES ARE LOCATED AT $-7/4\pm\sqrt{-39/4}$. THE NOISE SAMPLE IS THE SAME FOR BOTH CASES.

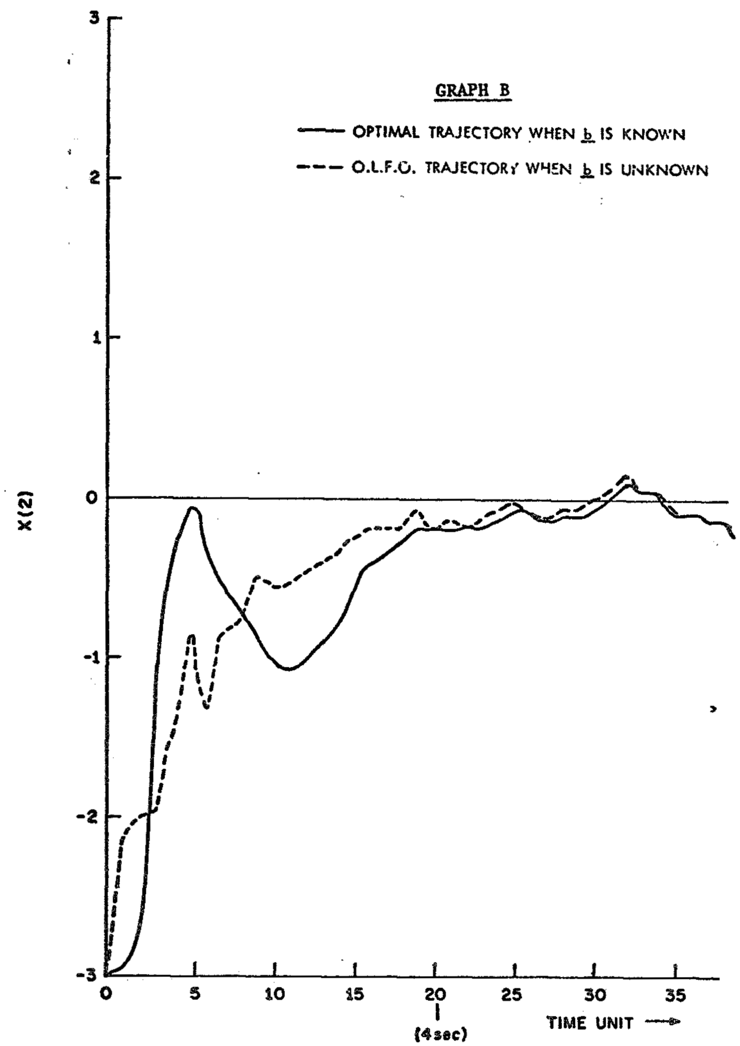


Fig. 6 (Continued)

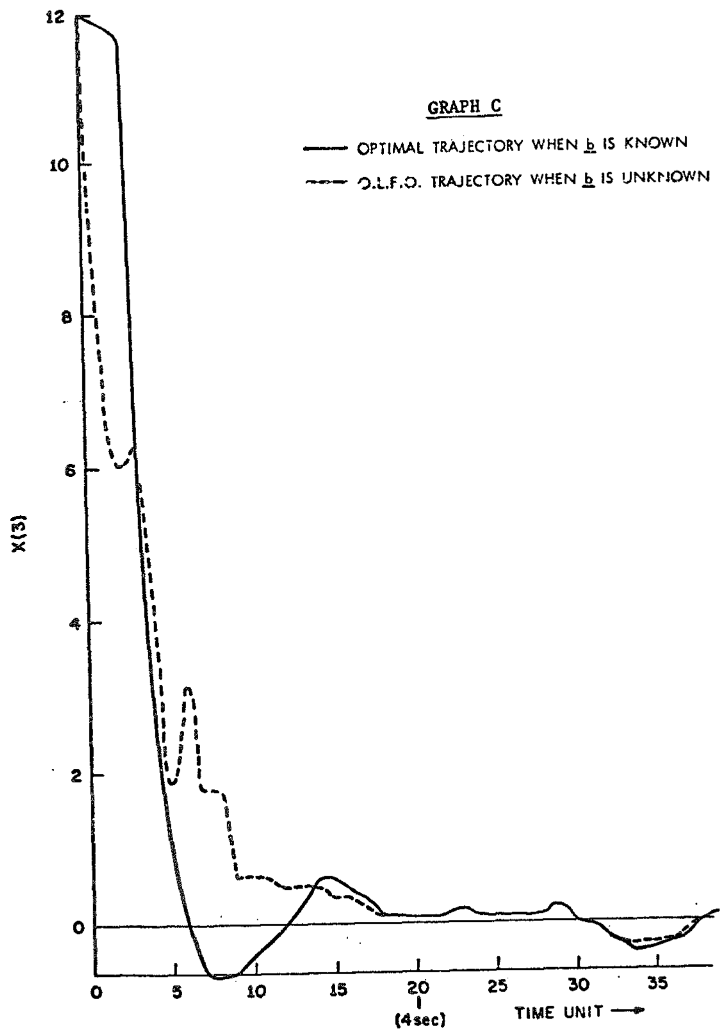


Fig. 6 (Continued)

Another set of simulation experiments was carried out where we kept the same sample noise but varied the weighting h, $(h > 0)$. It was found from the experiments (not reported in here) that the maximum magnitude of the overshoot in the O.L.F.O. trajectories varied inversely with the value of h; if h was large, we have relatively "lower" overshoots; whereas, if h was small, we had relatively high overshoots. Also, the experiments seem to indicate that the convergence rate and the final estimation error in $\underline{b}$ seem to depend on the value of h we chose; with large h, we have relatively slow convergence rate and relatively big final estimation error in $\underline{b}$; if h is small, we have a relatively fast convergence rate and relatively small final estimation error in $\underline{b}$.

In the next set of experiments, we kept the weighting fixed $(h = 0.1)$, and repeated the first set of experiments with larger driving noise covariance $(r = 0.45)$ while using the same observation noise sample. The experimental results (not reported in here) seem to indicate that the increase in driving noise covariance has little effect on the convergence rate of the O.L.F.O. control system.

It is of interest to find out whether the initial guess on b_f will be sensitive to the resulting O.L.F.O. control system. We carried out a set of experiments where we fixed

$$\underline{b}_f = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad ; \quad \underline{A}_f = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ .5 & -3 & -1 \end{bmatrix} \tag{10.17}$$

Thus the true transfer function is

$$H_1'(s) = \frac{-1}{(s - 1)(s^2 + 2s + s)} \tag{10.18}$$

The initial condition on $\underline{x}_f(0)$ was kept fixed, and using the same sample noise, we varied our initial guess in $\underline{b}_f$. The same runs seem to indicate that though the sample O.L.F.O. trajectory varied with different initial guesses in $\underline{b}_f$; the convergence rate was quite insensitive to the guess in $\underline{b}_f$.

Example 2: Stable System

It is assumed that

$$\underline{A}_f = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -7 & -3 \end{bmatrix} \; ; \quad \underline{b}_f = \begin{bmatrix} 1 \\ 2 \\ -7 \end{bmatrix} \; ; \quad \underline{x}_f(0) = \begin{bmatrix} 6 \\ -3 \\ 12 \end{bmatrix} \qquad (10.19$$

The true transfer function for the system is (Fig.2).

$$H_2(s) = \frac{(s+3)(s+2)}{(s+1)(s^2+2s+s)} \qquad (10.20$$

The system is stable.

In the first set of experiments, we initially guess

$$\hat{\underline{b}}_f(0/0) = \begin{bmatrix} 2 \\ 1 \\ -6 \end{bmatrix} \qquad (10.21)$$

i.e. that the zeroes are located at $-\frac{7}{4}+\sqrt{\frac{-39}{4}}$ and $-\frac{7}{4}-\sqrt{\frac{-39}{4}}$. The weighting on the control is $h = 1$. We take the final time $N = 40$.

Sample runs for the same system with same initial guess (10.21) were made and the plots for one particular sample are shown in Figs. 6, 7, 7. As opposed to the unstable case, the O.L.F.O. adaptive gain is some nonzero vector, and so the value of the O.L.F.O. control is not zero at the beginning (Fig. 8). This confirms the remarks made in Section 7. The control is used both for identification and

control purposes. The system is stable, and since no large magnitude control is applied, the O.L.F.O. trajectory decays down to zero (see Fig. 6). This decaying phenomenon is noticed by the identifier, and thus the control is kept near zero to save energy. Therefore, after a certain time interval, when the O.L.F.O. trajectory goes near the origin, the O.L.F.O. control will remain zero for most of the time. The system behaves almost like an input-free system. In fact, this is also what the truly optimum system will do. We note from Fig. 7 that the identification process of the unknown gain $\underline{b}$ stops at about $k = 20$, which is the approximate time unit when the O.L.F.O. state trajectory begins to stay around zero. If we consider control over an infinite interval (say using a window-shifting approach) we may expect awfully slow convergence rate in the estimation of $\underline{b}$ to the true $\underline{b}$, and a slow convergence rate of O.L.F.O. control system to truly optimum control system.

In the second set of experiments, we used the same noise samples as before but starting with the initial condition

$$\underline{x}_f(0) = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \qquad (10.22)$$

The initial guess on $\underline{b}_f$ was

$$\hat{\underline{b}}_f(0/0) = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix} \qquad (10.23)$$

i.e. the plant had no zeroes. The weighting on the control is $h = 1$, and we take the final time $N = 60$. The plots for one typical sample experiment are shown in Figs. 9, 10, 11. (The sample noise for the sample run shown in Figs. 9, 10, 11 is the same as that shown in Figs. 6,7,8. Comparing

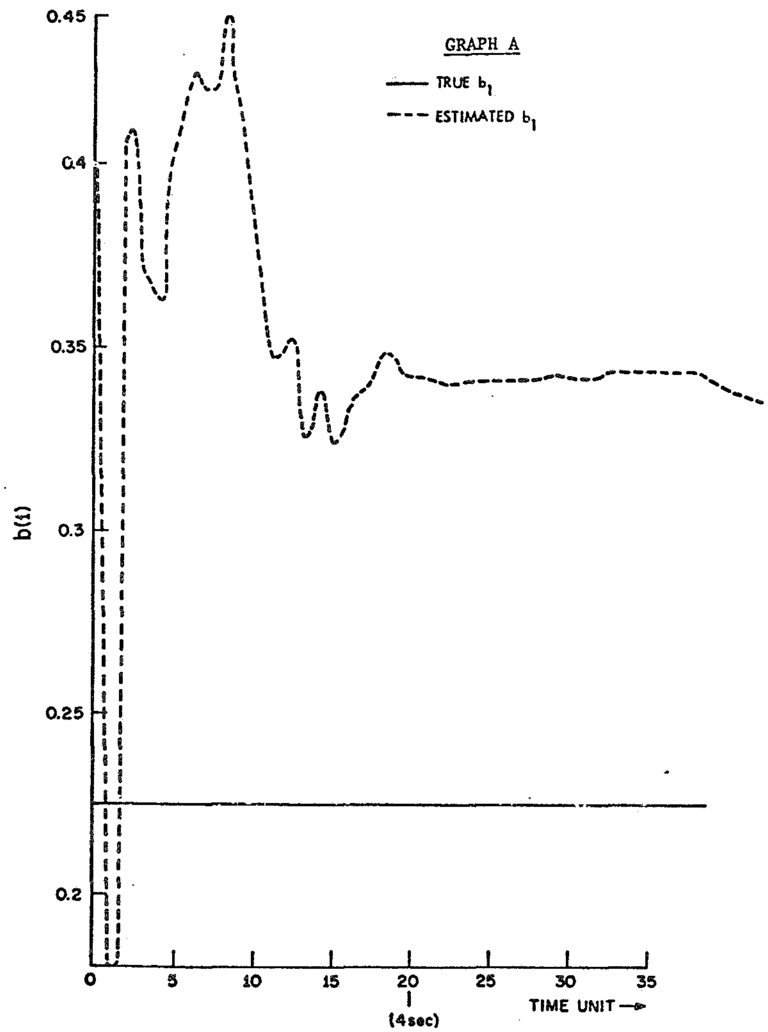


Fig. 7 ESTIMATE OF GAIN VECTOR. THE SYSTEM BEING CONSIDERED HAS SYSTEM FUNCTION $\frac{(S+3)\ (S+2)}{(S+1)\ (S^2+2S+5)}$. WE GUESS INITIALLY THAT THE ZEROES ARE LOCATED AT $-7/4\pm\sqrt{-39/4}$.

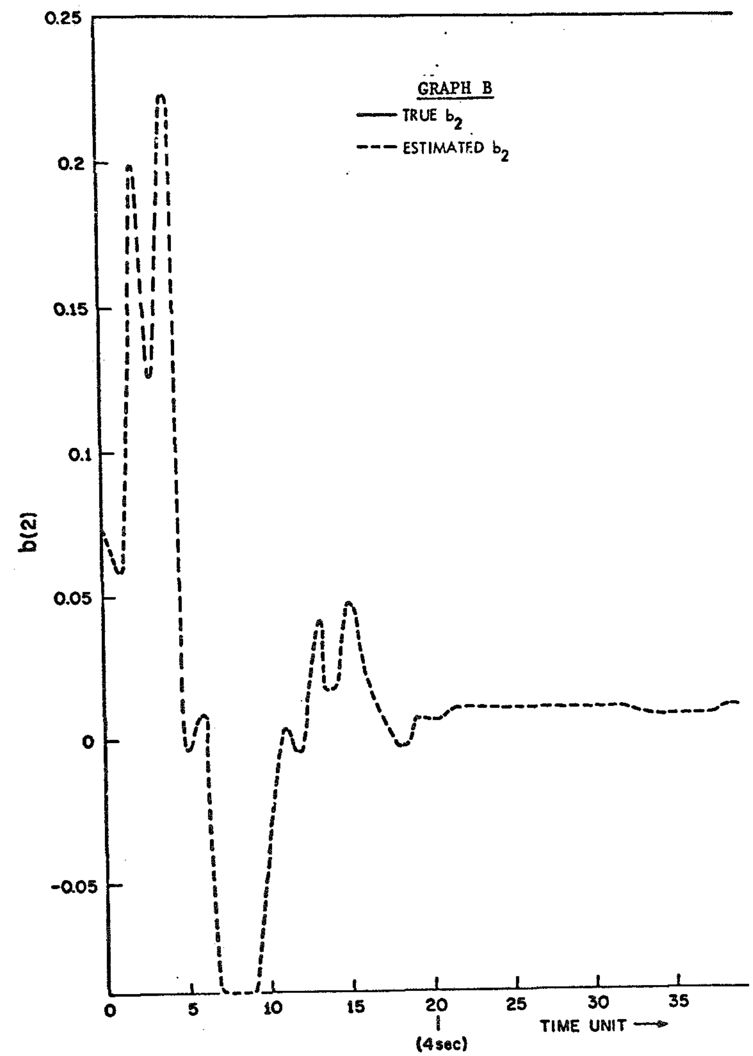


Fig. 7 (Continued)

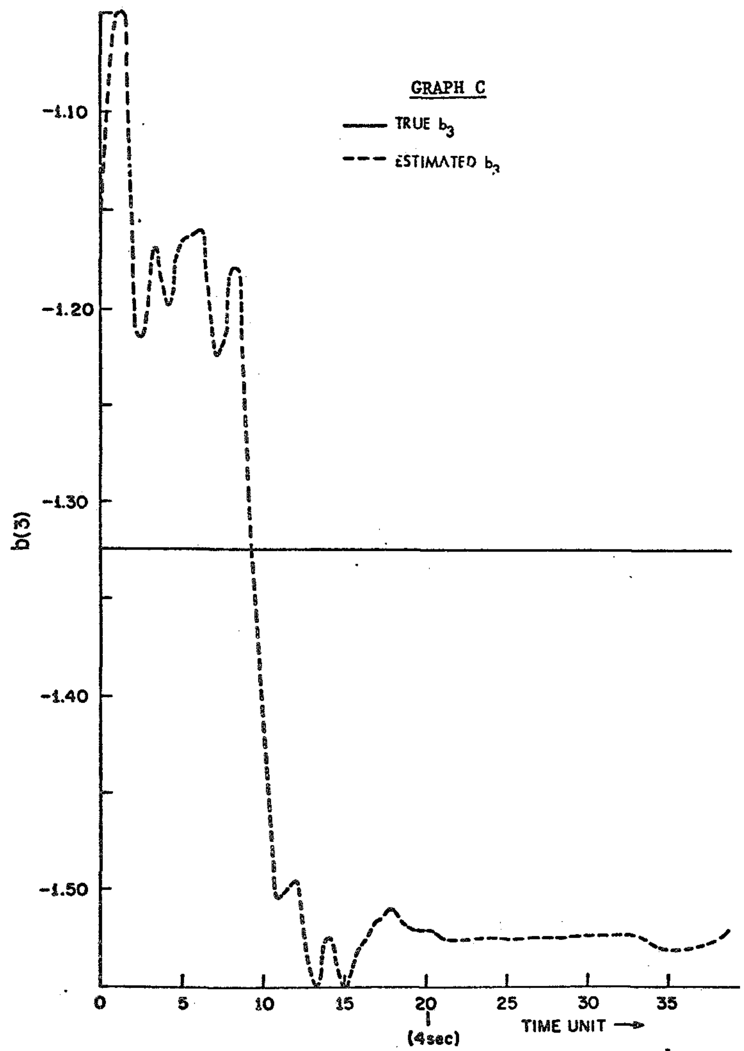


Fig. 7 (Continued)

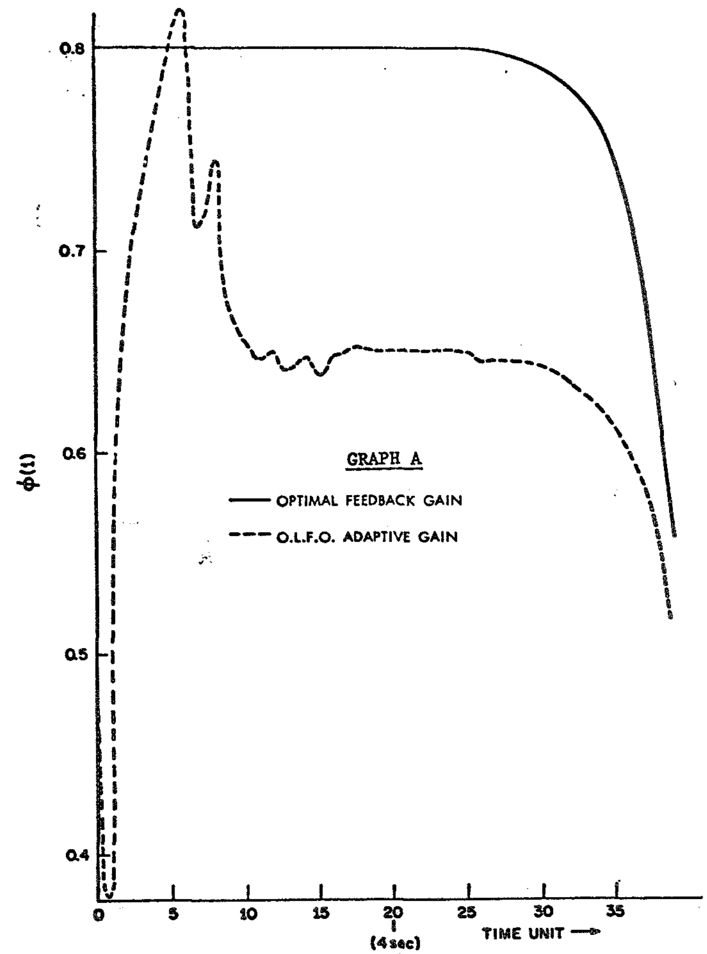


Fig. 8 COMPARISON BETWEEN OPTIMAL FEEDBACK GAIN AND O.L.F.O. ADAPTIVE GAIN. THE SYSTEM BEING CONSIDERED HAS SYSTEM FUNCTION $\frac{(S+3)(S+2)}{(S+1)(S^2+2S+5)}$. WE GUESS INITIALLY THAT THE ZEROES ARE LOCATED AT $-7/4 \pm \sqrt{-39/4}$

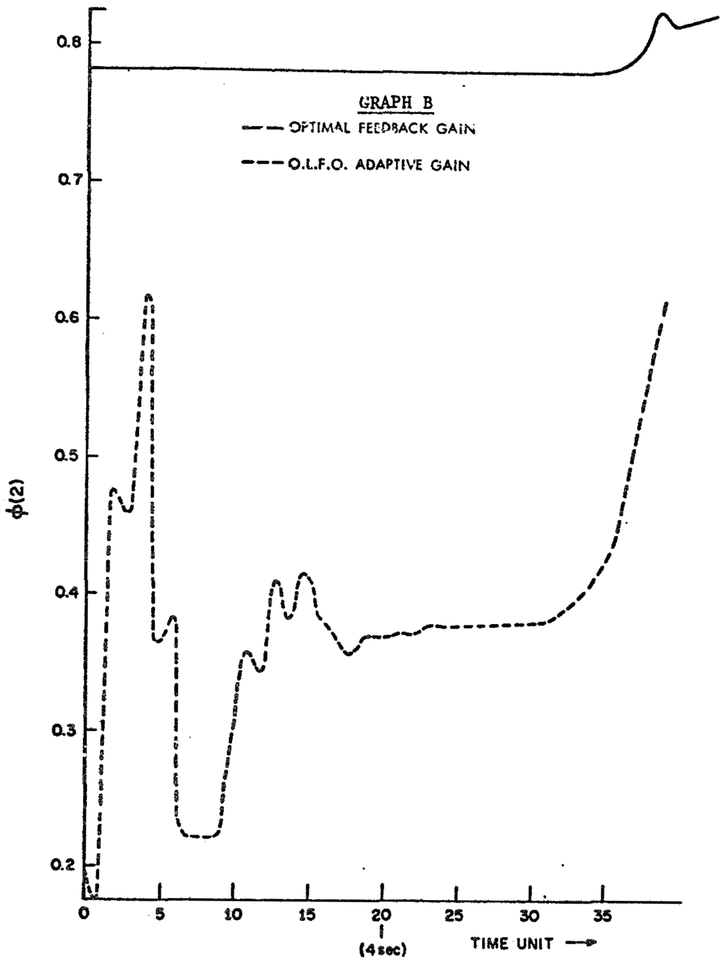


Fig. 8 (Continued)

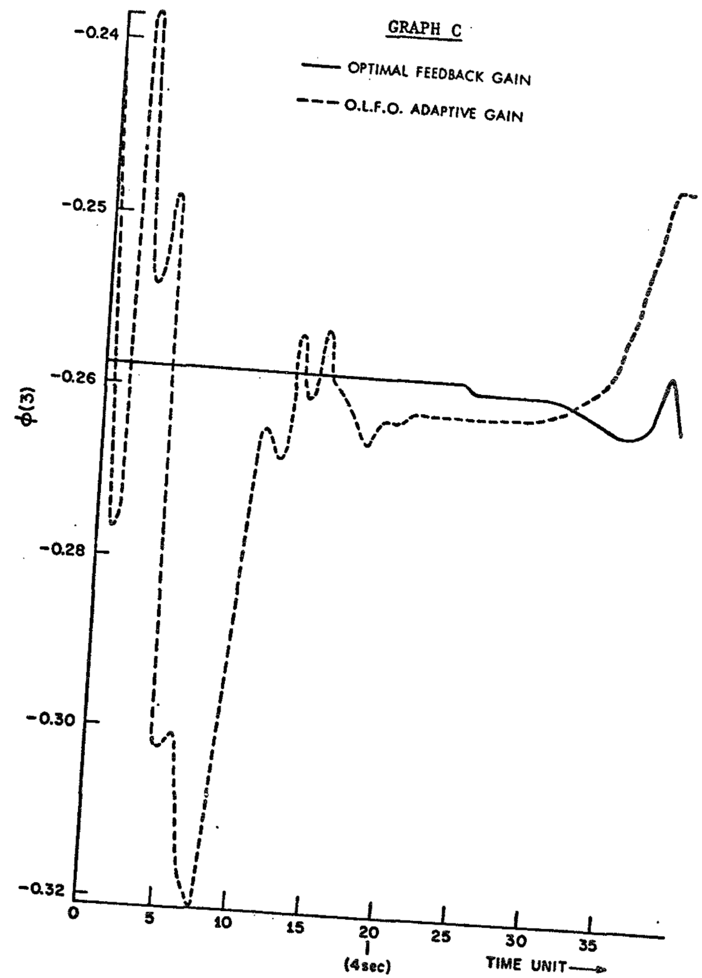


Fig. 8 (Continued)

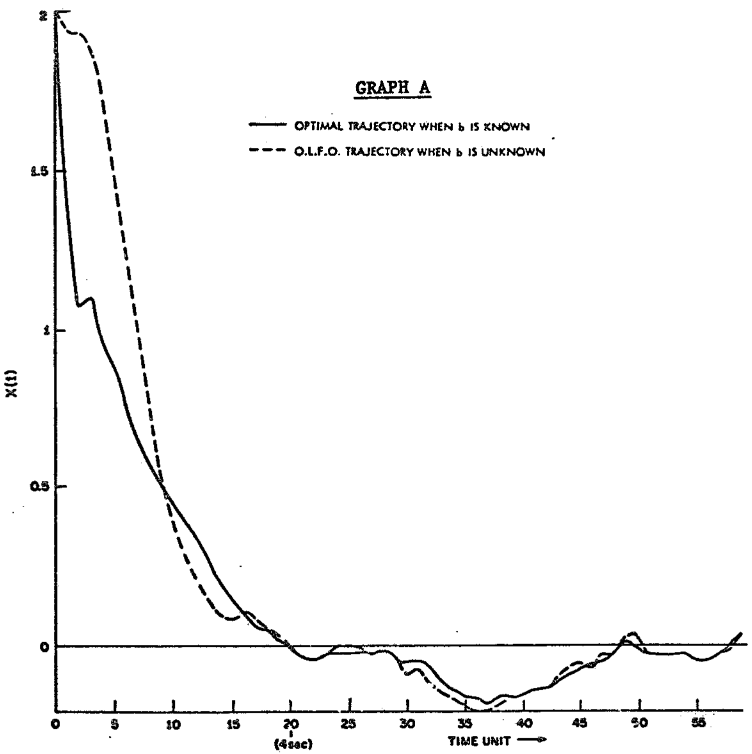


Fig. 9 COMPARISON BETWEEN OPTIMAL TRAJECTORY WHEN THE GAIN VECTOR IS KNOWN AND THE O.L.F.O. TRAJECTORY ASSUMING THE GAIN VECTOR IS UNKNOWN. THE SYSTEM BEING CONTROLLED HAS SYSTEM FUNCTION $\frac{(S+3)\ (S+2)}{(S+1)\ (S^2+2S+3)}$. WE GUESS INITIALLY THAT THERE ARE NO Z EROES. THE NOISE SAMPLE IS THE SAME FOR BOTH CASES.

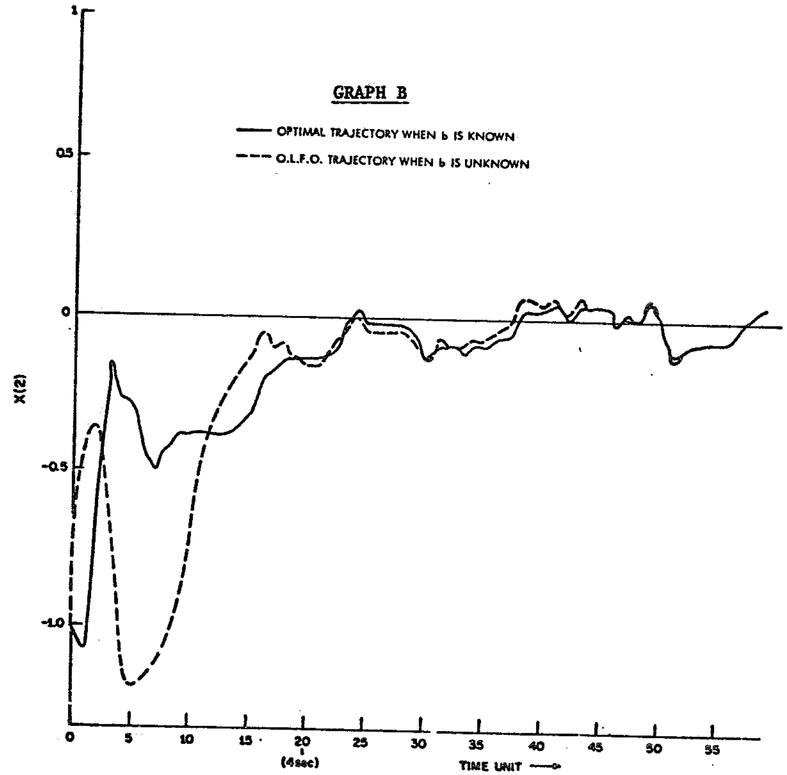


Fig. 9 (Continued)

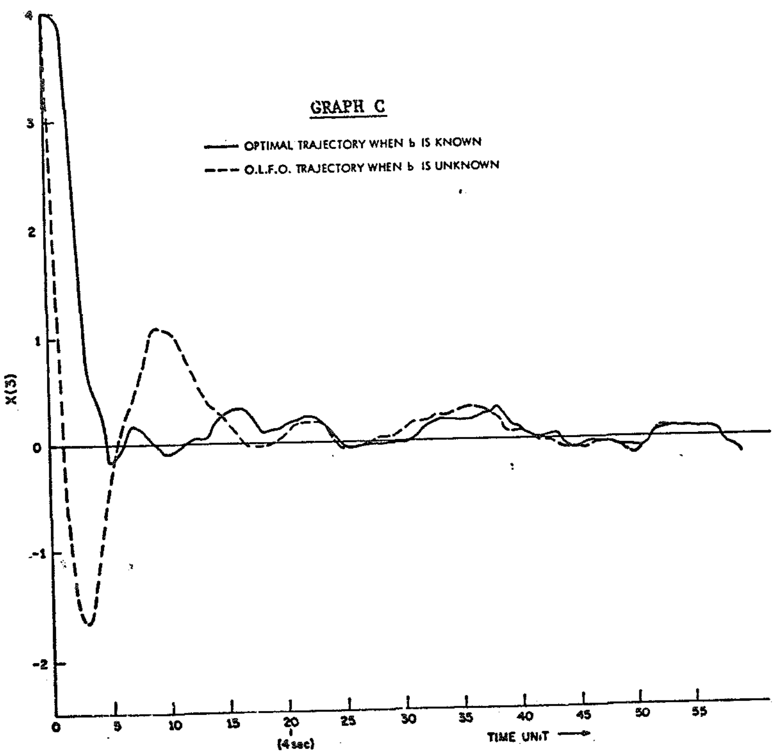


Fig. 9 (Continued)

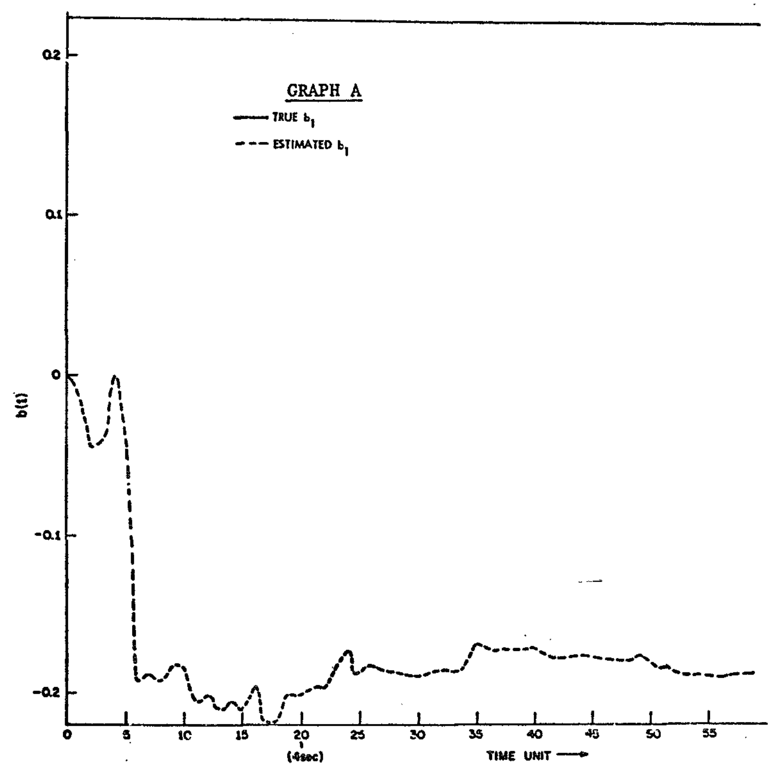


Fig. 10 ESTIMATE OF THE GAIN VECTOR. THE SYSTEM BEING CONSIDERED HAS SYSTEM FUNCTION $\frac{(S+3)\ (S+2)}{(S+1)\ (S^2+2S+3)}$. THE INITIAL GUESS IS THAT THERE ARE NO ZEROES.

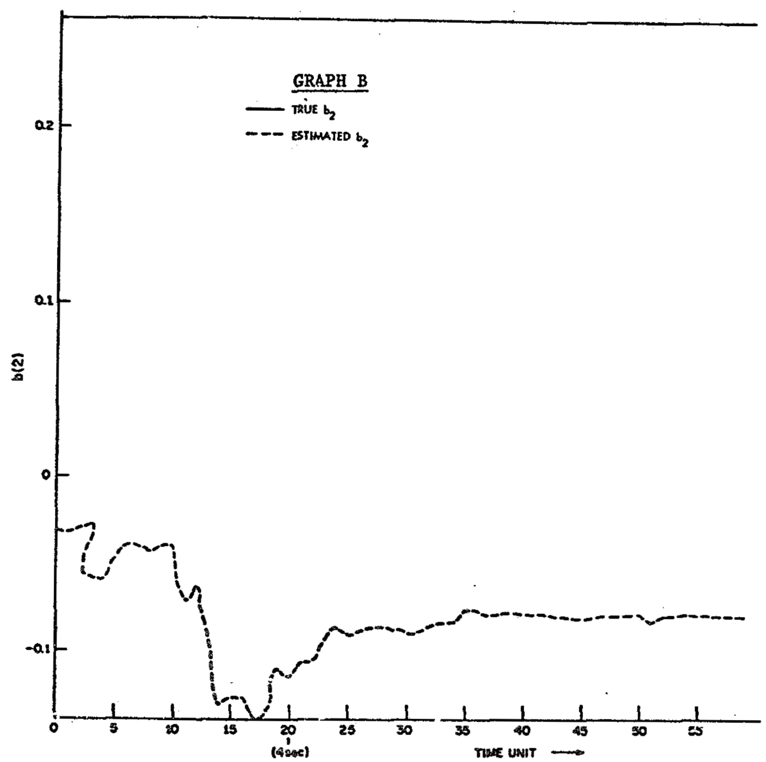


Fig. 10 (Continued)

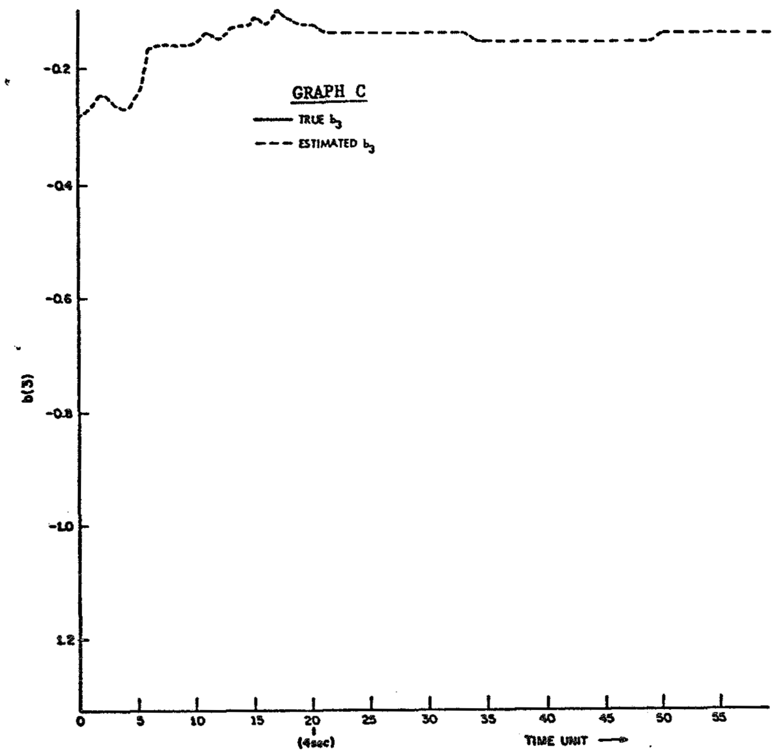


Fig. 10 (Continued)

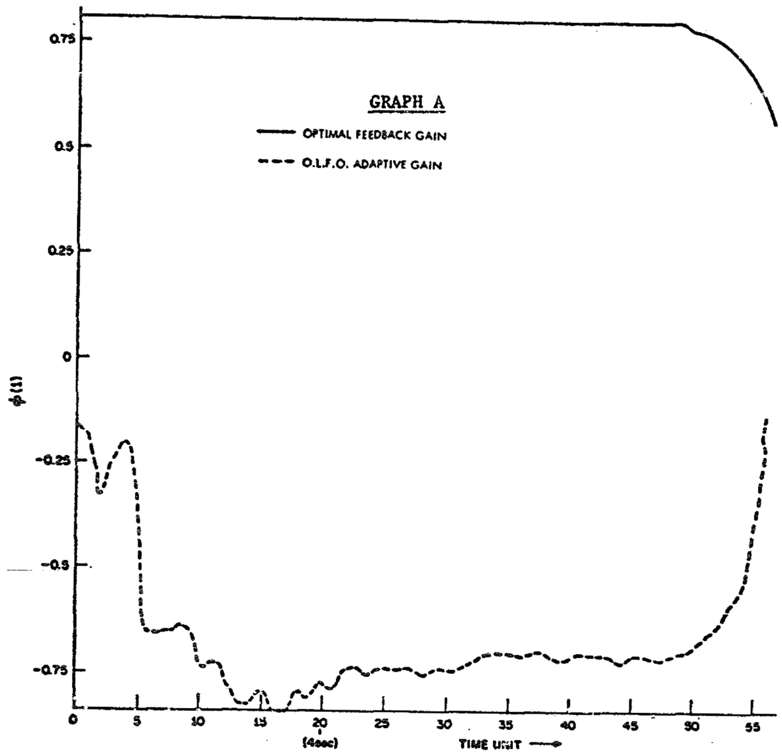


Fig. 11 COMPARISON BETWEEN OPTIMAL TO FEEDBACK GAIN AND O.L.F.O. ADAPTIVE GAIN. THE SYSTEM BEING CONSIDERED IS $\frac{(S+3)\ (S+2)}{(S+1)\ (S^2+2S+2)}$. THE INITIAL GUESS IS THAT THERE ARE NO ZEROES.

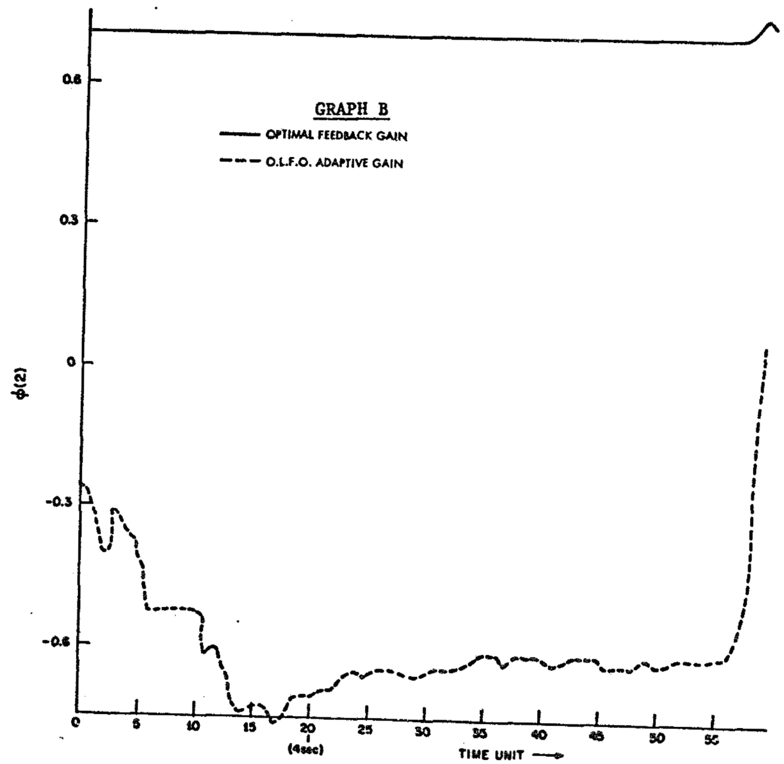


Fig. 11 (Continued)

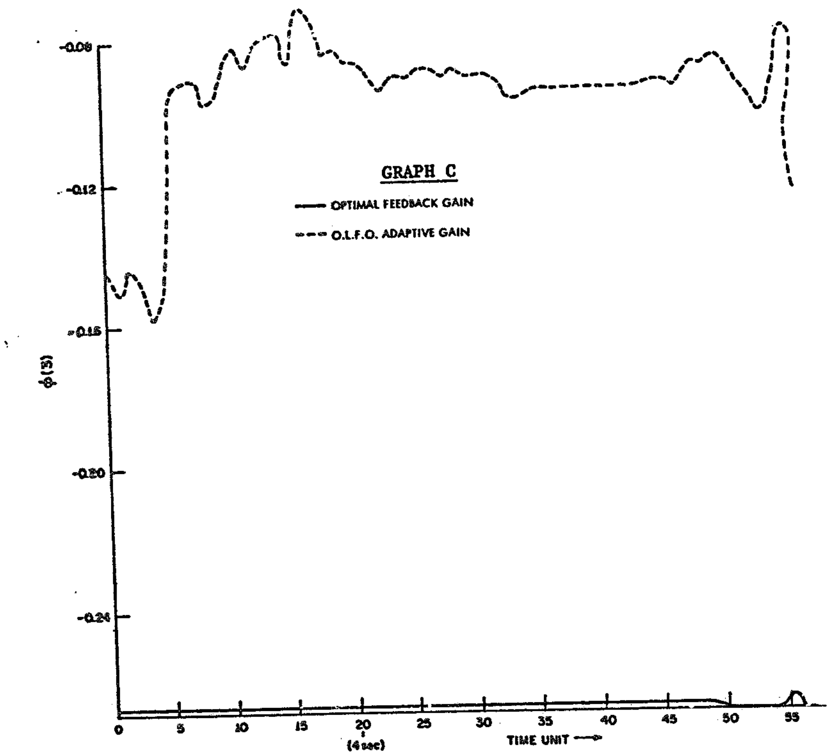


Fig. 11 (Continued)

this set of experiments with the last, we note that more or less the same phenomenon occurred in both sets of experiments. The final estimate in b is way off its true value, in fact $\hat{b}_1(k/k)$ and $\hat{b}_2(k/k)$ are opposite in sign with those of b_1 and b_2 respectively; but interestingly enough the adaptive gains are adjusted accordingly so that the values of the O.L.F.O. control sequence and the truly optimal control sequence are almost the same. This set of experiments indicates yet slower convergence (if there is any).

Note that in both sets of experiments even if the estimate of b does not converge to the true b, the truly optimal trajectory and O.L.F.O. trajectory are almost the same after the transient period.

Intuitively, the results are reasonable. Since we have not told the problem to identify b, it will not do so unless the identification is absolutely necessary as to conserve control energy. The experimental results verified our theoretical deduction of Section 7.

The experiments seem to indicate that for stable system, the choice of initial guess will not greatly influence the O.L.F.O. trajectory, but will affect the convergence rate for the estimate in the gain parameters, b.

Remark: In each set of experiments discussed above, the number of sample runs is not enough to enable us to draw specific statistical conclusions; yet the regularity in the sample runs enable us to draw some crude conclusions.

From the simulations, we may draw the following conclusions which agree with the theoretical predictions regarding the O.L.F.O. control system.

(1) The rate of convergence seems to be very dependent on the stability of the system. For unstable systems, the convergence rate seems to be faster compared to that for stable systems.

(2) It seems that large controls will help identification of the unknown gain parameters, and so convergence rate seems to relate directly to the magnitude of the control action.

(3) For unstable systems, the rate of convergence seems to be fairly independent of the initial guess on the unknown gain, whereas for stable systems, the convergence rate may be quite dependent on the initial guess on the unknown gain.

(4) For unstable systems, the O.L.F.O. trajectory will depend on the the initial guess in $\underline{b}_f$, but then for stable systems, the O.L.F.O. trajectory will not vary drastically when we vary the initial guess in $\underline{b}_f$.

(5) For the unstable system, the O.L.F.O. trajectory seems to follow closely its input-free trajectory in the beginning, until the diverging phenomenon tells the identifier to send back large controls for identification purposes. This causes some overshoots in the trajectory. The magnitude of the maximum overshoot seems to relate inversely with the values for the weighting constant h on control. For stable systems, simultaneous identification and control seem to be carried out in the beginning. Since the system is stable, with little control energy, the state will go to zero, so after some time period, when the state is near the origin, approximately zero control is applied thus terminating the identification of $\underline{b}$.

(6) Lastly, we sould like to comment on the computational feasibility of the proposed scheme. The above experiments were

simulated using an IBM 360/64/40 system. It was found that the actual computation of the O.L.F.O. control sequence can be carried out almost in real time for $N = 40$; i.e. in about 0.2 second, the following tasks were accomplished: One step computation of (4.19) - (4.23) (6 vector difference equation and 6×6 matrix difference equation), the parameter computations (5.3) - (5.6), and the computation of $\underline{\tilde{K}}(k|k)$ (5.2), $\underline{S}(k)$ (5.8) (one 12×12 matrix difference equation and one 3×3 matrix difference equation, computed in a time-backward direction directly for $k \leq 40$ steps, $k = 0,1,\ldots,N-1$).

11. CONCLUSIONS

A technique for adaptive control for a class of linear systems with unknown gain parameters has been presented. Simulation results have verified qualitative theoretical predictions.

The technique proposed is more general than that proposed by Farison et al[3] since they enforced separation so that the control gains are not adjusted by the uncertainty (covariance matrices) of the parameter estimates. It differs from that proposed by Murphy [4], Gorman and Zaborszky[5], Bar-Shalom and Sivan[6], and Florentin[7] in the sense that these stated or developed techniques to approximate Bellman's equation. The paper by Bar-Shalom and Sivan[6] did propose an O.L.F.O. approach to the problem but no detailed derivations were carried out; thus, one could not deduce qualitative properties of the adaptive system.

12. ACKNOWLEDGMENT

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APPENDIX C

PROOFS ON ASYMPTOTIC BEHAVIOR

Proof of Theorem 8.2

By (4.17) and (4.20), we have

$$\underline{M}^{!}_{\underline{A},\bar{\underline{C}}}(k,2\nu) = \left[\begin{array}{l:l} \underline{C}(k) & \underline{0} \\ \underline{C}(k+1)\underline{\phi}_A(k,k) & \underline{C}(k+1)u(k) \\ \vdots & \vdots \\ \underline{C}(k+j)\underline{\phi}_A(k+j-1,k) & \sum_{\ell=k}^{k+j+1} \underline{C}(k+j)\underline{\phi}_A(k+j-1,\ell+1)u(\ell)\underline{\phi}_G(\ell-1,k) \\ \vdots & \vdots \\ \underline{C}(k+2\nu-1)\underline{\phi}_A(k+2\nu-2,k) & \sum_{\ell=k}^{k+2\nu-2} \underline{C}(k+2\nu-1)\underline{\phi}_A(k+2\nu-2,\ell+1)u(\ell)\underline{\phi}_G(\ell-1,k) \end{array}\right] \tag{C.1}$$

By assumption, the first $m\nu$ rows of vectors contains at least n independent vectors. Among the rows vectors $\underline{C}(k + \nu + j)\underline{\phi}_A(k + \nu + j - 1,k)$, let

$\underline{c}'_{\rho_j(1)}(k + \nu + j)\underline{\phi}_A(k + \nu + j - 1,k)$, ... $\underline{c}'_{\rho_j(\nu_j)}(k + \nu + j)\underline{\phi}_A(k + \nu + j - 1,k)$,

be the ν_j vectors which are independent of the row vectors:

$\underline{C}(k + \nu)\underline{\phi}_A(k + \nu - 1,k)$, $\underline{C}(k + \nu - 1)\underline{\phi}_A(k + \nu,k)$, ... $\underline{C}(k + \nu + j - 1)$ $\underline{\phi}_A(k + \nu + j - 2,k)$, $j = 1, \ldots, \nu - 1$; where

$$\underline{C}(k + \nu + j) = \begin{bmatrix} \underline{c}'_1(k + \nu + j) \\ \vdots \\ \underline{c}'_m(k + \nu + j) \end{bmatrix} \tag{C.2}$$

and $\rho_j(\cdot)$ is some permutation of $\{1, 2, \ldots, m\}$. Since $\{(\underline{A}(k),\underline{C}(k))\}_{k=0}^{\infty}$ is uniformly completely observable of index ν, it follows that $\nu_i \neq 0$, $i = 1, \ldots, \nu - 1$, and that

$$m + \nu_1 + \nu_2 + \ldots \nu_{\nu-1} = n \quad . \tag{C.3}$$

Assume that we have the dependence

$$\underline{c}'_{\rho_j(s)}(k+\nu+j)\underline{\phi}_A(k+\nu+j-1,k) = \sum_{i=0}^{\nu+j-1} \underline{\alpha}'_i(j,s)\underline{C}(k+i)\underline{\phi}_A(k+i-1,k); \quad 1 \leq s \leq \nu_j \tag{C.4}$$

where the only possible nonzero entries of $\underline{\alpha}'_i(j,s)$, $i = 0, \ldots, \nu + j - 1$, are those corresponding to independent rows of $\underline{C}(k + i)\underline{\phi}_A(k + i - 1,k)$, $i = 0, \ldots, \nu + j - 1$. If there exists no $\underline{\alpha}'_i(j,s)$, $i = 0, \ldots, \nu + j - 1$, which bears the relation (C.4), then the $(m(\nu + j - 1) + \rho(s))$th row vector of $\underline{M}^{!}_{\underline{A},\bar{\underline{C}}}(k,2\nu)$ is independent of the first $m(\nu + j - 1)$ row vectors. If there exists $\underline{\alpha}'_i(j,s)$, $i = 0, \ldots, \nu + j - 1$ which gives the dependence (C.4), then such a dependence is unique by construction. Now assume that the $(m(\nu + j - 1) + \rho(s))$th row vector of $M^{!}_{\underline{A},\bar{\underline{C}}}(k,2\nu)$ is dependent on the first $m(\nu + j - 1)$ row vectors, then we must also have the dependence

$$\sum_{\ell=k}^{k+\nu+j-1} \underline{c}'_{\rho_j(s)}(k+\nu+j)\underline{\phi}_A(k+\nu+j-1,\ell+1)u(\ell)\underline{\phi}_G(\ell-1,k) =$$
$$\sum_{i=1}^{\nu+j-1} \underline{\alpha}'_i(j,s)\sum_{\ell=k}^{k+s-1} \underline{C}(k+i)\underline{\phi}_A(k+i-1,\ell+1)u(\ell)\underline{\phi}_G(\ell-1,k) \quad . \tag{C.5}$$

Since $\underline{A}(k)$ is nonsingular, by (C.4) we have

$$\sum_{i=0}^{r+j-1} \sum_{\ell=k+i}^{k+\nu+j-1} \underline{\alpha}'_i(j,s)\underline{C}(k+i)\underline{\psi}_A(k+i-1,\ell+1)u(\ell)\underline{\phi}_G(\ell-1,k) = \underline{0}_n \tag{C.6}$$

where

$$\underline{\psi}_A(i,j) = \underline{A}^{-1}(i)\underline{A}^{-1}(i + 1) \ldots \underline{A}^{-1}(j) \quad ; \quad i > j \quad . \tag{C.7}$$

Since $\{(\underline{A}(k),\underline{C}(k))\}_{k=0}^{\infty}$ is uniformly completely observable, the vector

$$\underline{\alpha}^{j\,'}(j,s) \triangleq [\underline{\alpha}'_0(j,s) \ \cdots \ \vdots \ \underline{\alpha}'_j(j,s)] \tag{C.8}$$

cannot be the zero row vector, $s = 1, \ldots, \nu_j$. By assumption $\underline{G}(k)$ is nonsingular, therefore (C.6) is true if and only if $u(k+i) = 0$, $i = 0, 1, \ldots, j$ which is a contradiction. This result applies for $s = 1, \ldots, \nu_j$; $j = 0, 1, \ldots, \nu - 1$. Together with (C.3) and the remark made at the beginning of the proof, we have that $M_{\tilde{A},\tilde{C}}(k,2\nu)$ will have rank $2n$ if $u(k+i) \neq 0$, $i = 0, 1, \ldots, \nu - 1$. The theorem follows from the assumption that $u(k) \neq 0$, $k = 0, 1, \ldots$.

Proof of lemma 8.4

From (4.23) and (4.21), since $N(k) = 0$, we have

$$\Sigma_b(k+1|k+1,U(0,k)) = G(k)\Sigma_b(k|k,U(0,k-1)G'(k) - [0 \vdots I_n]V^*(k+1|k,U(0,k))$$
$$(\tilde{C}(k+1)\tilde{A}(k|k,U(0,k))\tilde{C}'(k+1)+Q(k+1))V^{*\prime}(k+1|k,U(0,k))\begin{bmatrix} 0 \\ \cdots \\ I_n \end{bmatrix} \qquad \text{(C.9)}$$

where $V^*(k+1|k,U(0,k))$ satisfies (4.21) – (4.23), using (8.2), (8.3) follows immediately from (C.9).

Proof of Theorem 8.5 (Main Result)

Let $\varepsilon > 0$ such that

$$||\Sigma_b(k+2\nu|k+2\nu,U(0,k+2\nu-1)) - \Sigma_b(k|k,U(0,k-1))|| \leq \varepsilon \qquad \text{(C.10)}$$

where $||\cdot||$ is the spectral norm. Since $\Sigma_b(k|k,U(0,k-1)) \geq 0$, $k = 0, 1, \ldots$, (8.3) and (C.10) imply that we have the inequality

$$||\Sigma_b(k+j|k+j,U(0,k+j-1) - \Sigma_b(k+j-1|k+j-1,U(0,k+j-2)|| \leq \varepsilon$$
$$j = 1, 2, \ldots, 2\nu \quad . \qquad \text{(C.11)}$$

Using equation (C.9), we have

$$\varepsilon \geq ||[0 \vdots I_n]V^*(k+j|k+j-1,U(0,k+j-1)\cdot(\tilde{C}(k+j)\tilde{A}(k+j-1|k+j-1,U(0,k+j-1))\cdot$$
$$\tilde{C}'(k+j) + Q(k+j))V^{*\prime}(k+j|k+j-1,U(0,k+j-1))\begin{bmatrix} 0 \\ \cdots \\ I_n \end{bmatrix}|| \quad . \qquad \text{(C.12)}$$

By corollary (8.3), $\tilde{C}(k+j)\tilde{A}(k+j-1|k+j-1,U(0,k+j-1))\tilde{C}'(k+j) + Q(k+j)$ can be uniformly bounded, so

$$||(\tilde{C}(k+j)\tilde{A}(k+j-1|k+j-1,U(0,k+j-1))\tilde{C}'(k+j)+Q(k+j))V^{*\prime}(k+j|k+j-1,U(0,k+j-1))\begin{bmatrix} 0 \\ \cdots \\ I_n \end{bmatrix}|| \leq$$
$$||(\tilde{C}(k+j)\tilde{A}(k+j-1|k+j-1,U(0,k+j-1)\tilde{C}'(k+j)+Q(k+j))^{\frac{1}{2}}||\cdot||(\tilde{C}(k+j)\tilde{A}(k+j-1|k+j-1,U(0,k+j-1))$$
$$\tilde{C}'(k+j)+Q(k+j))^{\frac{1}{2}}V^*(k+j|k+j-1,U(0,k+j-1))\begin{bmatrix} 0 \\ \cdots \\ I_n \end{bmatrix}||$$
$$\leq \alpha_j \sqrt{\varepsilon} \triangleq \delta_j(\varepsilon) \qquad j = 1, 2, \ldots, 2\nu \qquad \text{(C.13)}$$

$\delta_j(\varepsilon)$ is continuous in ε and $\delta_j(\varepsilon) \to 0$ as $\varepsilon \to 0$, $j = 1, \ldots, \nu$. Using (4.21), (C.13) can also be written as follows

$$||C(k+j)A(k+j-1)\Sigma_{xb}(k+j-1|k+j-1,U(0,k+j-2))G'(k+j-1)+u(k+j-1)C(k+j)\cdot$$
$$\Sigma_b(k+j-1|k+j-1,U(0,k+j-2))G'(k+j-1)|| \leq \delta_j(\varepsilon)$$
$$j = 1, \ldots, 2\nu \quad . \qquad \text{(C.14)}$$

Since $V^*(k+j|k+j-1,U(0,k+j-1))$ is bounded for $j = 1, \ldots, 2$, therefore (C.12) and (4.21) imply that

$$||[0 \vdots I_n]V^*(k+j|k+j-1,U(0,k+j-1))\tilde{C}(k+j)\tilde{A}(k+j-1|k+j-1,U(0,k+j-1))\begin{bmatrix} 0 \\ \cdots \\ I_n \end{bmatrix}|| \leq \beta_1(\varepsilon) \qquad \text{(C.15)}$$

$$||[I_n \vdots 0]V^*(k+j|k+j-1,U(0,k+j-1))\tilde{C}(k+j)\tilde{A}(k+j-1|k+j-1,U(0,k+j-1))\begin{bmatrix} 0 \\ \cdots \\ I_n \end{bmatrix}||$$
$$\leq \beta_2(\varepsilon) \qquad \text{(C.16)}$$

where $\beta_i(\varepsilon)$ is continuous in ε, $\beta_i(\varepsilon) \to 0$ as $\varepsilon \to 0$, $i = 1, 2, \ldots$. By using

(4.23), (C.15) and (C.16) and the assumption that $\underline{G}(k)$ is nonsingular, the inequality (C.14) implies

$$\left\| [\underline{C}(k+1 \; \vdots \; \underline{0}] \begin{bmatrix} \Sigma_{xb}(k+1|k+1,U(0,k)) \\ \\ \Sigma_b(k+1|k+1,U(0,k)) \end{bmatrix} \right\| \le f_1(\epsilon) \qquad (C.17)$$

$$\left\| [\underline{C}(k+j)\phi_A(k+j-1,k+1) \; \vdots \; \underline{C}(k+j) \sum_{\ell=k+1}^{k+j-1} \phi_A(k+j-1,\ell+1)u(\ell)\phi_G(\ell-1,k+1)] \begin{bmatrix} \Sigma_{xb}(k+1|k+1,U(0,k)) \\ \\ \Sigma_b(k+1|k+1,U(0,k)) \end{bmatrix} \right\| \le f_j(\epsilon)$$

$$j = 2, 3, \ldots, 2\nu \qquad (C.18)$$

where $f_i(\epsilon)$ is continuous in ϵ, $f_i(\epsilon) \to 0$ as $\epsilon \to 0$, $i = 1, 2, \ldots, 2\nu$. Equations (C.17) and (C.18) imply

$$\left\| \underline{M}_{\tilde{A},\tilde{C}}(k+1,2\nu) \begin{bmatrix} \Sigma_{xb}(k+1|k+1,U(0,k)) \\ \\ \Sigma_b(k+1|k+1,U(0,k)) \end{bmatrix} \right\| \le f(\epsilon) \qquad (C.19)$$

where $f(\epsilon) \to 0$ when $\epsilon \to 0$ and is continuous in ϵ. By theorem 8.2, $\underline{M}_{\tilde{A},\tilde{C}}(k+1,2\nu)$ is of full rank, so we have

$$\| \Sigma_{xb}(k+1|k+1,U(0,k)) \| \le \delta'(\epsilon) \qquad \delta'(\epsilon) \to 0 \quad \text{as } \epsilon \to 0 \qquad (C.20)$$

$$\| \Sigma_b(k+1|k+1,U(0,k)) \| \le \delta''(\epsilon) \qquad \delta''(\epsilon) \to 0 \quad \text{as } \epsilon \to 0 \; . \qquad (C.21)$$

Now the conclusion of the theorem follows from (8.4).

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13. ABSTRACT

The problem considered in this two-part paper deals with the control of linear, discrete-time, stochastic cystems with unknown (possibly time-varying and random) gain parameters. The philosophy of control is based on the use of an open-loop-feedback-optimal (O.L.F.O.) control using a quadratic index of performance. In Part I it is shown that the O.L.F.O. system consists of (1) an identifier that estimates the system state variables and gain parameters, and (2) by a controller described by an "adaptive" gain and correction term. Several qualitative properties of the overall system are obtained from an interpretation of the equations. Part II deals with the asymptotic properties of the O.L.F.O. adaptive system and with simulation results dealing with the control of stable and unstable third order plants. Comparisons are carried out with the optimal system when the parameters are known. In addition, the simulation results are interpreted in the context of the qualitative conclusions reached in Part I.

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